The edit distance on graphs, Part III

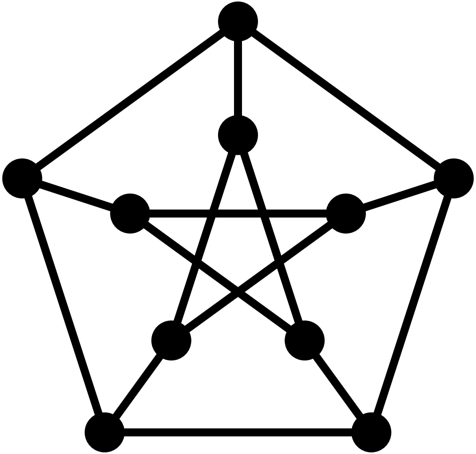
Ryan Martin

Iowa State University

5th Lake Michigan Workshop on Combinatorics and Graph Theory
Notre Dame University

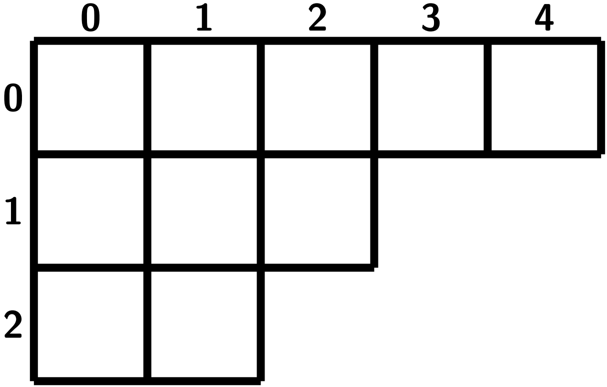
Survey: The edit distance in graphs: methods, results and generalizations, *Recent Trends in Combinatorics*, 31–62, IMA Vol. Math. Appl., **159**, Springer, Cham, 2016.

Petersen!



$$\mathcal{H} = \mathrm{Forb}(P_{10})$$

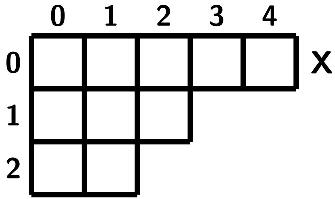
Clique spectrum

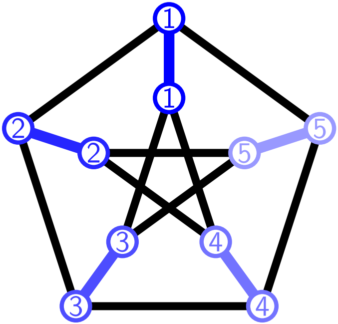


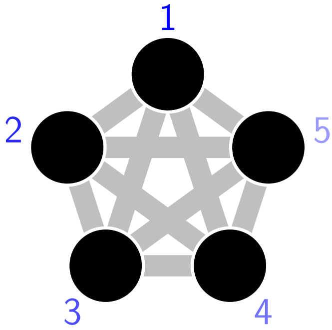
Clique Spectrum of $\mathrm{Forb}(P_{10})$

$(0,5) \notin \Gamma$

$\Gamma(\mathrm{Forb}(P_{10}))$:





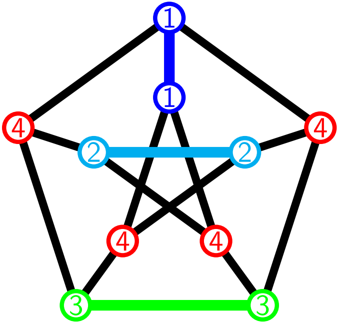


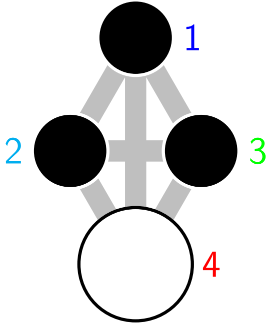
$(0,5) \notin \Gamma$

$(1,3) \notin \Gamma$

$\Gamma(\mathrm{Forb}(P_{10}))$:

	0	1	2	3	4
0					
1				X	
2					

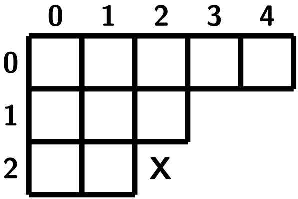


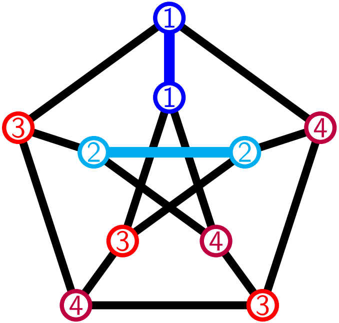


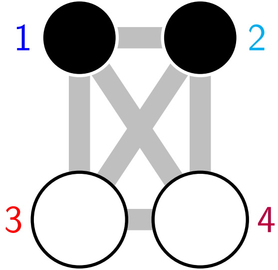
$$(1,3) \notin \Gamma$$

$(2,2) \notin \Gamma$

$\Gamma(\mathrm{Forb}(P_{10}))$:

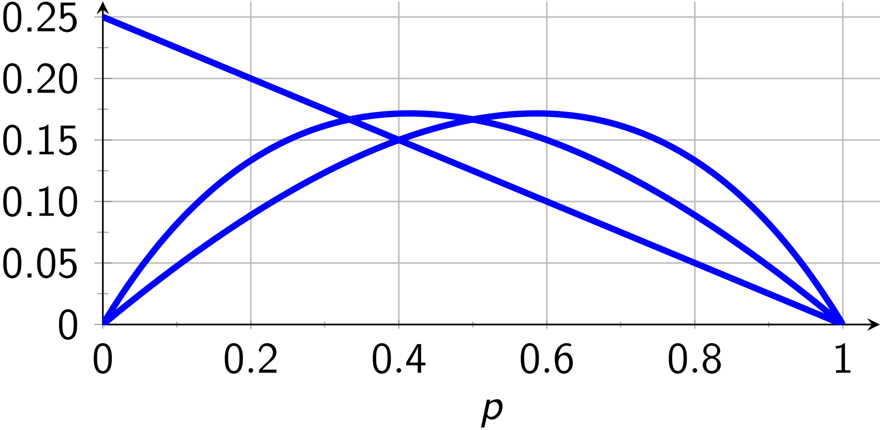






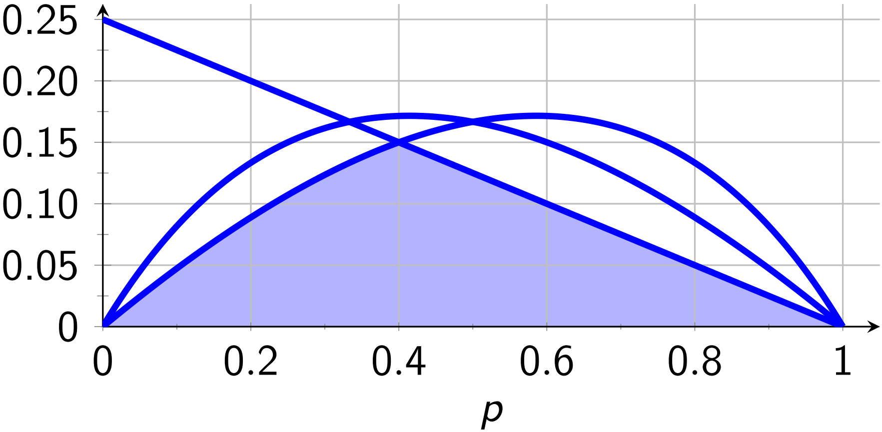
$(2,2) \notin \Gamma$

$\gamma_{\mathcal{H}}(p)$



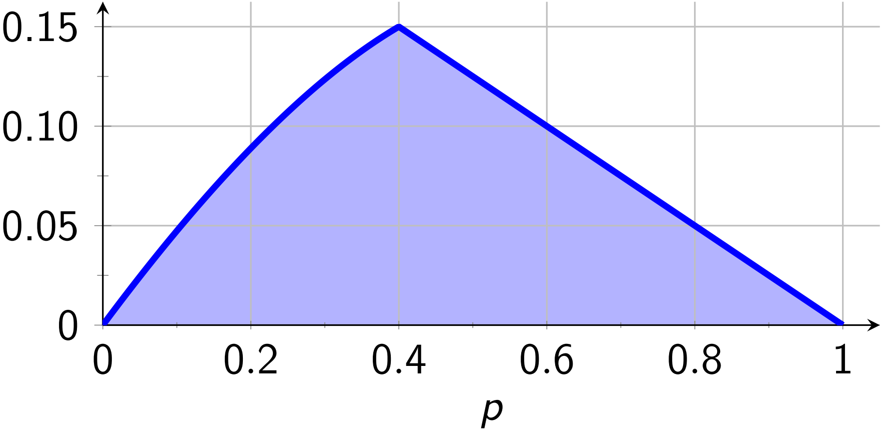
Plot of $\frac{p(1-p)}{2-p}$, $\frac{p(1-p)}{1+p}$, and $\frac{1-p}{4}$.

$\gamma_{\mathcal{H}}(p)$



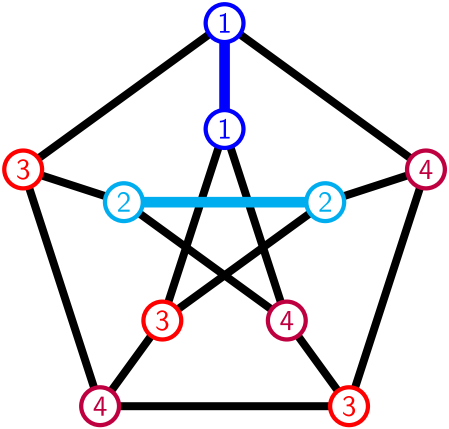
Plot of $\frac{p(1-p)}{2-p}$, $\frac{p(1-p)}{1+p}$, and $\frac{1-p}{4}$.

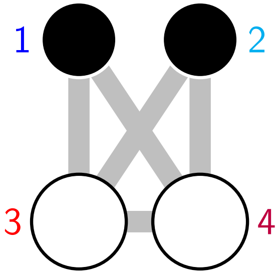
$\gamma_{\mathcal{H}}(p)$



Plot of $\gamma_{\mathcal{H}}(p) = \min\left\{\frac{p(1-p)}{2-p}, \frac{1-p}{4}\right\}$.

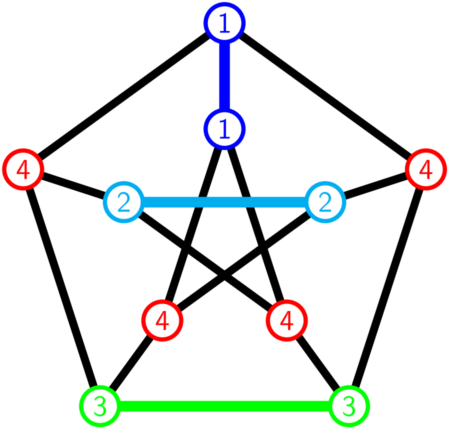
$|\mathrm{VW}| = 2 \implies |\mathrm{VB}| \leq 1$

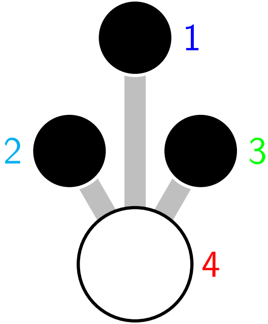




$$|\mathrm{VW}| = 2 \implies |\mathrm{VB}| \leq 1$$

$|\mathrm{VW}| = 1 \implies |\mathrm{VB}| \leq 2$





$$|\mathrm{VW}| = 1 \implies |\mathrm{VB}| \leq 2$$

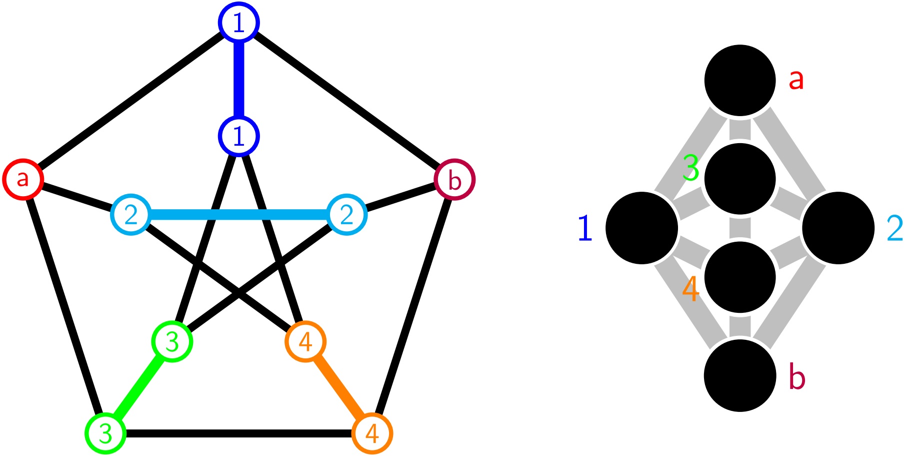
$|\mathrm{VW}| = 2 \implies |\mathrm{VB}| \leq 1$

If $p \geq 1/2$, then $g_{\mathcal{H}}(p) = \dfrac{1-p}{4}$.

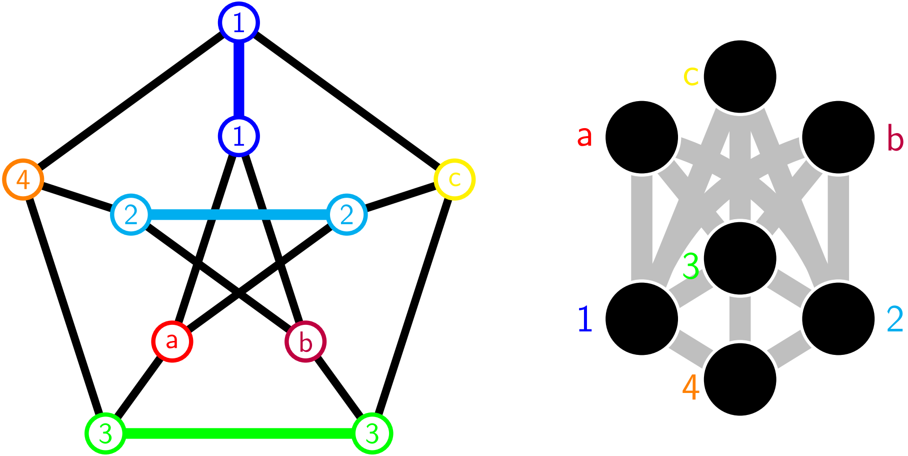
If $p < 1/2$, then we may assume all vertices v are in VB and

$$d_G(v) = \frac{p-g}{p} + \frac{1-2p}{p}\mathbf{x}(v).$$

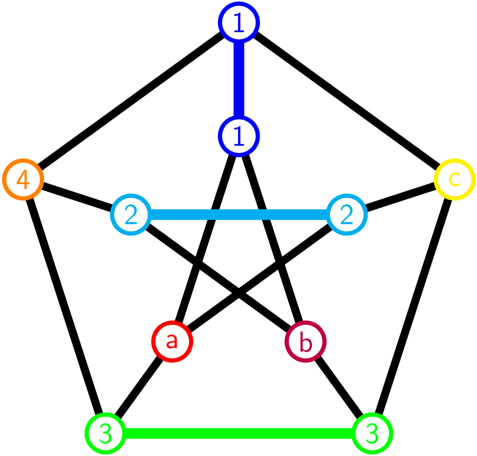
Neighbors of a football

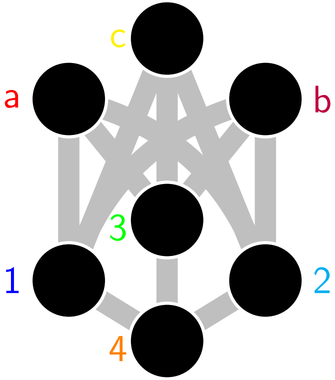


No $K_{3,4}$

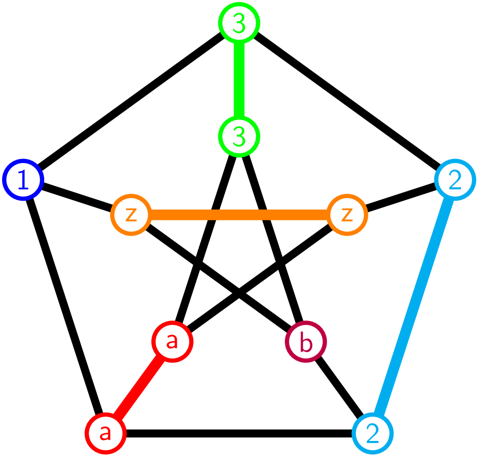


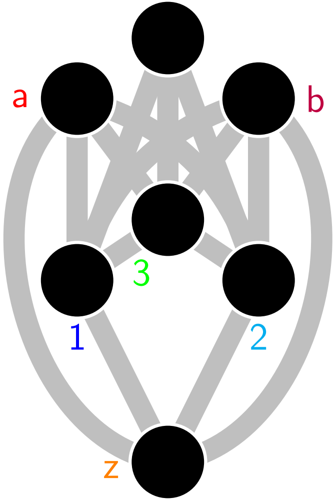
No $K_{3,4}$



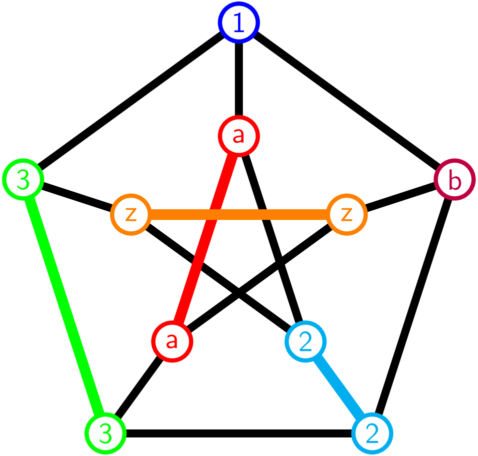


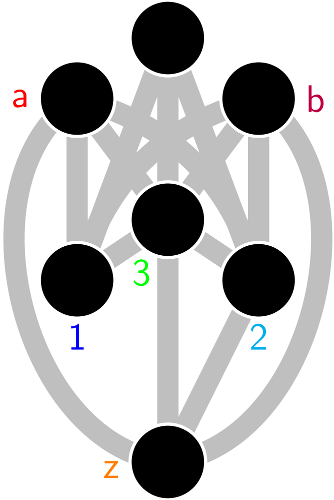
No 4 neighbors, first case



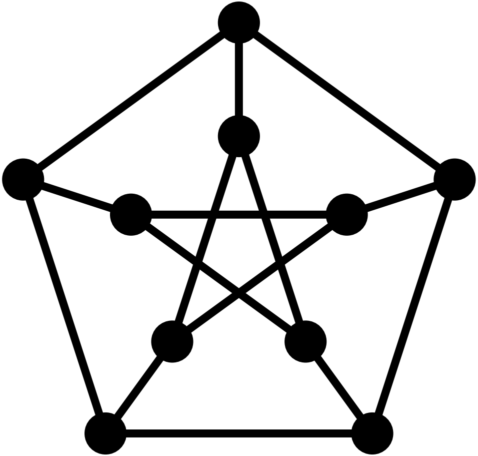


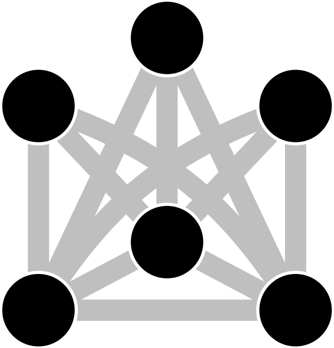
No 4 neighbors, second case





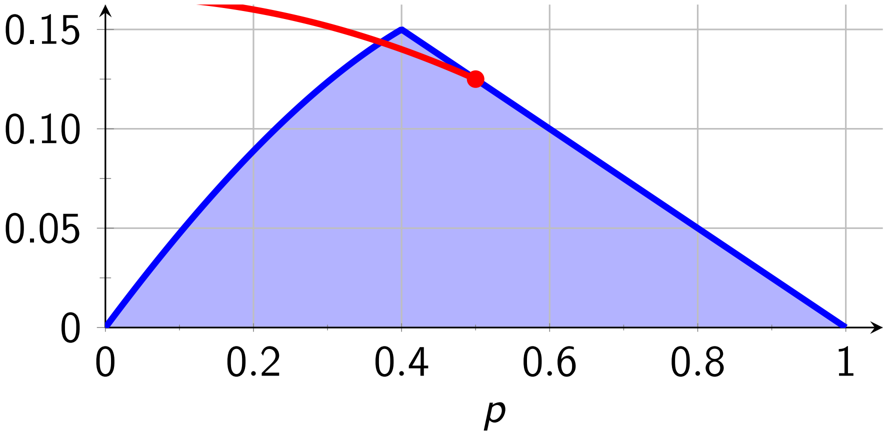
New CRG



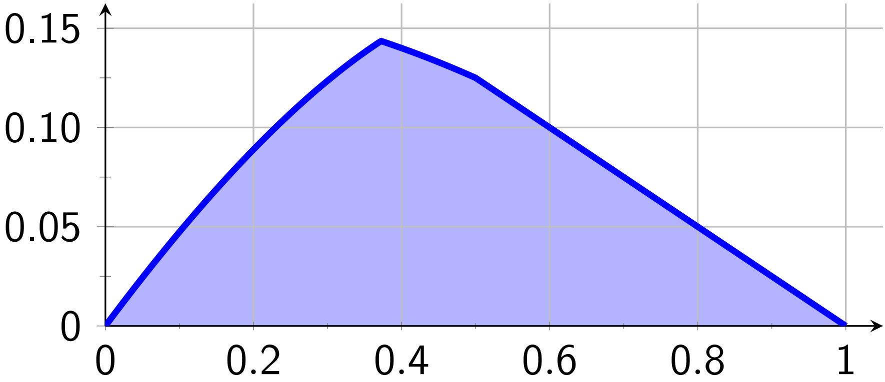


$$g_K(p) = \frac{1-p^2}{6}$$

Revised plot



Zoomed plot



$$\mathrm{ed}_{\mathrm{Forb}(P_{10})}(p) = \min\left\{\frac{p(1-p)}{2-p}, \frac{1-p^2}{6}, \frac{1-p}{4}\right\}$$

$$p^*_{\mathrm{Forb}(P_{10})} = \frac{\sqrt{33}-5}{2} \approx 0.372 \qquad d^*_{\mathrm{Forb}(P_{10})} = \frac{5\sqrt{33}-27}{12} \approx 0.144$$

Future work

- More edit distance functions. What do they tell us about hereditary properties?

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?
- Directed graphs and multicolorings of complete graphs.

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    Weak and strong k-ary chromatic numbers.

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    Weak and strong k-ary chromatic numbers.
- Other combinatorial structures: hypergraphs

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    Weak and strong k-ary chromatic numbers.
- Other combinatorial structures: hypergraphs
  - Problem: The edit distance function is well-defined for hypergraphs, but hypergraph Turán numbers are a special case of the edit distance function.

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    Weak and strong k-ary chromatic numbers.
- Other combinatorial structures: hypergraphs
  - Problem: The edit distance function is well-defined for hypergraphs, but hypergraph Turán numbers are a special case of the edit distance function.
- Graph limits. In this setting, it is more natural to investigate other metrics besides the edit metric.

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    Weak and strong k-ary chromatic numbers.
- Other combinatorial structures: hypergraphs
  - Problem: The edit distance function is well-defined for hypergraphs, but hypergraph Turán numbers are a special case of the edit distance function.
- Graph limits. In this setting, it is more natural to investigate other metrics besides the edit metric.
- Connections to classic combinatorial problems.

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?

Conjecture

Fix $p_0 \in [0,1]$ and let $H \sim G(n_0, p_0)$ with $\mathcal{H} = \mathrm{Forb}(H)$. Then,

$$\mathrm{ed}_{\mathcal{H}}(p) \sim \frac{2\log_2 n_0}{n_0} \min\left\{ \frac{p}{\log_2 \frac{1}{1-p_0}}, \frac{1-p}{\log_2 \frac{1}{p_0}} \right\}.$$

with probability $\to 1$ as $n_0 \to \infty$.

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?

Conjecture

Fix $p_0 \in [0,1]$ and let $H \sim G(n_0, p_0)$ with $\mathcal{H} = \mathrm{Forb}(H)$. Then,

$$\mathrm{ed}_{\mathcal{H}}(p) \sim \frac{2\log_2 n_0}{n_0} \min\left\{ \frac{p}{\log_2 \frac{1}{1-p_0}}, \frac{1-p}{\log_2 \frac{1}{p_0}} \right\}.$$

with probability $\to 1$ as $n_0 \to \infty$.

$$p^*_{\mathcal{H}} \sim \frac{\log(1-p_0)}{\log(p_0(1-p_0))}.$$

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\mathrm{ed}_{\mathrm{Forb}(G(n_0,p_0))}(p)$?

Conjecture

Fix $p_0 \in [0,1]$ and let $H \sim G(n_0,p_0)$ with $\mathcal{H} = \mathrm{Forb}(H)$. Then,

$$\mathrm{ed}_{\mathcal{H}}(p) \sim \frac{2\log_2 n_0}{n_0} \min\left\{ \frac{p}{\log_2 \frac{1}{1-p_0}}, \frac{1-p}{\log_2 \frac{1}{p_0}} \right\}.$$

with probability $\to 1$ as $n_0 \to \infty$.

$$p^*_{\mathcal{H}} \sim \frac{\log(1-p_0)}{\log(p_0(1-p_0))}.$$

Counterintuitive because $\frac{\log(1-p_0)}{\log(p_0(1-p_0))} = p_0$ if and only if $p_0 \in \left\{0, \frac{1}{2}, 1\right\}$.

Thanks!

- The 5th Lake Michigan Workshop on Combinatorics and Graph Theory
  - David Galvin
  - Patrick Bennett
  - Andrej Dudek
  - University of Notre Dame
  - National Science Foundation (NSF)
  - Institute for Mathematics and its Applications (IMA)
- Financial Support: NSF, IMA, National Security Agency (NSA), Simons Foundation
- Co-authors
  - Maria Axenovich
  - József Balogh
  - Zhanar Berikkyzy
  - André Kézdy
  - Tracy McKay
  - Chelsea Peck