The edit distance on graphs, Part III

Ryan Martin

Iowa State University

5th Lake Michigan Workshop on Combinatorics and Graph Theory
Notre Dame University

1. The problem

2. Clique Spectrum and $\gamma_H(p)$

3. Computing $g_H(p)$

4. Thanks!
\[ \mathcal{H} = \text{Forb}(P_{10}) \]
Clique Spectrum of $\text{Forb}(P_{10})$
$(0, 5) \notin \Gamma$

$\Gamma(\text{Forb}(P_{10}))$: 

```
  0 1 2 3 4  
0 0 0 0 0 0   
1 0 0 0 0 X   
2 0 0 0 0 0   
```

$(0, 5) \notin \Gamma$
(1, 3) ∉ Γ

Γ(Forb(P_{10})): 

\( (1, 3) \notin \Gamma \)
$(2, 2) \not\in \Gamma$

$\Gamma(\text{Forb}(P_{10})):$

$(2, 2) \not\in \Gamma$
Plot of $\frac{p(1-p)}{2-p}$, $\frac{p(1-p)}{1+p}$, and $\frac{1-p}{4}$.
Plot of $\frac{p(1-p)}{2-p}$, $\frac{p(1-p)}{1+p}$, and $\frac{1-p}{4}$.
Plot of $\gamma_{\mathcal{H}}(p) = \min \left\{ \frac{p(1-p)}{2-p}, \frac{1-p}{4} \right\}$. 

$\gamma_{\mathcal{H}}(p)$
$|VW| = 2 \implies |VB| \leq 1$
\[ |V_W| = 1 \implies |V_B| \leq 2 \]
If $p \geq 1/2$, then $g_{\mathcal{H}}(p) = \frac{1-p}{4}$.

If $p < 1/2$, then we may assume all vertices $v$ are in $\mathcal{V}_{B}$ and

$$d_G(v) = \frac{p - g}{p} + \frac{1 - 2p}{p} x(v).$$
Neighbors of a football
No $K_{3,4}$
No $K_{3,4}$
No 4 neighbors, first case
No 4 neighbors, second case

[Image of a graph with nodes labeled 1, 2, 3, a, b, z and connections between them.]

[Another image with a different style of graph and nodes labeled a, b, 1, 2, 3, z.]
New CRG

\[ g_K(p) = \frac{1 - p^2}{6} \]
Revised plot
\[ \text{ed}_{\text{Forb}}(P_{10})(p) = \min \left\{ \frac{p(1 - p)}{2 - p}, \frac{1 - p^2}{6}, \frac{1 - p}{4} \right\} \]

\[ p_{\text{Forb}}^*(P_{10}) = \frac{\sqrt{33} - 5}{2} \approx 0.372 \]

\[ d_{\text{Forb}}^*(P_{10}) = \frac{5\sqrt{33} - 27}{12} \approx 0.144 \]
Future work

- More edit distance functions. What do they tell us about hereditary properties?
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - \( \text{ed}_{\text{Forb}}(G(n_0,p_0))(p) \)?
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $ed_{Forb(G(n_0,p_0))}(p)$?
- Directed graphs and multicolorings of complete graphs.
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\text{ed}_{\text{Forb}}(G(n_0,p_0))(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - \( \text{ed}_{\text{Forb}}(G(n_0,p_0))(p) \)?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues: Weak and strong \( k \)-ary chromatic numbers.
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\text{ed}_{\text{Forb}(G(n_0,p_0))}(p)$?

- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    - Weak and strong $k$-ary chromatic numbers.

- Other combinatorial structures: hypergraphs
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\text{ed}_{\text{Forb}}(G(n_0,p_0))(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    - Weak and strong $k$-ary chromatic numbers.
- Other combinatorial structures: hypergraphs
  - Problem: The edit distance function is well-defined for hypergraphs, but hypergraph Turán numbers are a special case of the edit distance function.
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\text{ed}_{\text{Forb}}(G(n_0,p_0))(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    Weak and strong $k$-ary chromatic numbers.
- Other combinatorial structures: hypergraphs
  - Problem: The edit distance function is well-defined for hypergraphs, but hypergraph Turán numbers are a special case of the edit distance function.
- Graph limits. In this setting, it is more natural to investigate other metrics besides the edit metric.
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $ed_{\text{Forb}}(G(n_0,p_0))(p)$?
- Directed graphs and multicolorings of complete graphs.
  - The theory goes through, though CRGs are more complex.
  - The binary chromatic number has analogues:
    Weak and strong $k$-ary chromatic numbers.
- Other combinatorial structures: hypergraphs
  - Problem: The edit distance function is well-defined for hypergraphs, but hypergraph Turán numbers are a special case of the edit distance function.
- Graph limits. In this setting, it is more natural to investigate other metrics besides the edit metric.
- Connections to classic combinatorial problems.
Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\text{ed}_{\text{Forb}}(G(n_0, p_0))(p)$?

Conjecture

Fix $p_0 \in [0, 1]$ and let $H \sim G(n_0, p_0)$ with $\mathcal{H} = \text{Forb}(H)$. Then,

$$
\text{ed}_{\mathcal{H}}(p) \sim \frac{2 \log_2 n_0}{n_0} \min \left\{ \frac{p}{\log_2 \frac{1}{1-p_0}}, \frac{1-p}{\log_2 \frac{1}{p_0}} \right\}.
$$

with probability $\to 1$ as $n_0 \to \infty$. 

Future work

- More edit distance functions. What do they tell us about hereditary properties?
  - $\text{ed}_{\text{Forb}(G(n_0,p_0))}(p)$?

Conjecture

Fix $p_0 \in [0, 1]$ and let $H \sim G(n_0, p_0)$ with $\mathcal{H} = \text{Forb}(H)$. Then,

$$\text{ed}_{\mathcal{H}}(p) \sim \frac{2 \log_2 n_0}{n_0} \min \left\{ \frac{p}{\log_2 \frac{1}{1-p_0}}, \frac{1-p}{\log_2 \frac{1}{p_0}} \right\}.$$  

with probability $\to 1$ as $n_0 \to \infty$.

$$p^*_H \sim \frac{\log(1-p_0)}{\log(p_0(1-p_0))}.$$
Future work

- More edit distance functions. What do they tell us about hereditary properties?
- $\text{ed}_{\text{Forb}}(G(n_0, p_0))(p)$?

Conjecture

Fix $p_0 \in [0, 1]$ and let $H \sim G(n_0, p_0)$ with $\mathcal{H} = \text{Forb}(H)$. Then,

$$\text{ed}_\mathcal{H}(p) \sim \frac{2 \log_2 n_0}{n_0} \min \left\{ \frac{p}{\log_2 \frac{1}{1-p}}, \frac{1-p}{\log_2 \frac{1}{p}} \right\}.$$  

with probability $\to 1$ as $n_0 \to \infty$.

$$p^*_\mathcal{H} \sim \frac{\log(1 - p_0)}{\log(p_0(1 - p_0))}.$$  

Counterintuitive because $\frac{\log(1-p_0)}{\log(p_0(1-p_0))} = p_0$ if and only if $p_0 \in \{0, \frac{1}{2}, 1\}$.  

Thanks!

- The 5th Lake Michigan Workshop on Combinatorics and Graph Theory
  - David Galvin
  - Patrick Bennett
  - Andrej Dudek
  - University of Notre Dame
  - National Science Foundation (NSF)
  - Institute for Mathematics and its Applications (IMA)

- Financial Support: NSF, IMA, National Security Agency (NSA), Simons Foundation

- Co-authors
  - Maria Axenovich
  - József Balogh
  - Zhanar Berikkyzy
  - André Kézdy
  - Tracy McKay
  - Chelsea Peck