

Problem 8. Joanne buys a new car and is thrilled to see the odometer reads 000000 when she gets in. She decides on a savings plan linked to the car's odometer. She decides that for each time a digit on the car's odometer changes, she will put 1 cent in her savings account. For example, as the odometer goes from 000298 to 000301, the changes are

$$000298 \rightarrow 000299 \rightarrow 000300 \rightarrow 000301,$$

and she must deposit $1 + 3 + 1 = 5$ cents. (When the odometer changes to 000299 to 000300, each of the last three odometer digits changes, adding 3 cents to her deposit.) Joanne drives the car for years and one day sees the odometer cycle from 654320 back to 654321. At this point, how much money has Joanne saved?

Solution 8. Joanne will have saved \$7,270.21 when her odometer hits 654321.

Every time the units digit changes, Joanne's savings increase by 1 cent. In going from 000000 to 654321 the unit's digit will change 654321 times. The 10's digit changes every 10 miles, so changes

$$\left\lfloor \frac{654321}{10} \right\rfloor = 65432 \quad \text{times.}$$

Repeating this reasoning for the other four digits we conclude that Joanne's savings, in cents, are

$$\begin{aligned} & \left\lfloor \frac{654321}{10^0} \right\rfloor + \left\lfloor \frac{654321}{10^1} \right\rfloor + \left\lfloor \frac{654321}{10^2} \right\rfloor + \left\lfloor \frac{654321}{10^3} \right\rfloor + \left\lfloor \frac{654321}{10^4} \right\rfloor + \left\lfloor \frac{654321}{10^5} \right\rfloor \\ &= 654321 + 65432 + 6543 + 654 + 65 + 6 = 727021 \quad \text{cents,} \end{aligned}$$

that is, \$7,270.21.