Problem 7. Every Saturday morning, Bob and Alice, both of whom love bike riding, each leave their houses and ride towards each other at their own constant speeds until they meet. If they leave their houses at the same time, then they meet in 2 hours. If Bob leaves his house one-half of $\frac{1}{2}$ an hour *after* Alice leaves, then they meet 1 hour and 48 minutes after Bob leaves. One Saturday morning Alice is ill and cannot ride. Bob decides to ride his bike to her house to visit her. How much time does it take Bob to ride from his house to Alice's?

Solution 7. It takes Bob 3 hours and 20 minutes to ride to Alice's house.

Let D be the distance in miles between the two houses, let v_a be Alice's riding speed, and let v_b be Bob's, both in miles per hour. When they are riding towards each other the distance between them is changing (decreasing) at a rate of $v_a + v_b$ miles per hour. After riding for half of an hour Alice covers $\frac{1}{2}v_a$ miles, so when Bob leaves, the distance between them is $D - \frac{1}{2}v_a$ miles. Therefore

$$\frac{D}{v_a + v_b} = 2 \text{ hours} \qquad \text{and} \qquad \frac{D - \frac{1}{2}v_a}{v_a + v_b} = 1 \text{ hour and } 48 \text{ minutes} = \frac{9}{5} \text{ hours.}$$

Subtracting the first equation form the second leads to

$$\frac{\frac{1}{2}v_a}{v_a + v_b} = \frac{1}{5} \quad \text{and then to} \quad v_a = \frac{2}{3}v_b.$$

Therefore

$$2 = \frac{D}{v_a + v_b} = \frac{D}{\frac{2}{3}v_b + v_b} = \frac{3}{5}\frac{D}{v_b}.$$

Hence the time it takes for Bob to ride to Alice's house is

$$\frac{D}{v_b} = \frac{10}{3} \text{ hours.}$$