Problem 4. Find *all* positive integers n with the following two properties:

i. The leading (e.g. leftmost) digit of n is 9.

ii. When the leading 9 is erased, the resulting integer (with one fewer digits) is $\frac{1}{37}$ of n.

Solution 4. The positive integers with this property are 925×10^k, $k = 0, 1, 2, \ldots$.

Suppose n is an $m+1$ digit number. Then $n = 9 \times 10^m + \ell$, where ℓ is an m digit number, possibly with one or more leading zeros. By condition ii., we have

$$9 \times 10^m + \ell = 37\ell \qquad \text{from which} \qquad 10^m = \frac{37\ell - \ell}{9} = 4\ell.$$

Therefore

$$\ell = \frac{10^m}{4} = 2^{m-2}5^n = 25 \times 10^{m-2}.$$

Because ℓ is a positive integer, $m \geq 2$, so $\ell = 25 \times 10^k$, for $k = 0, 1, 2, \ldots$. Thus the collection of ns satisfying i. and ii. are of the form 925×10^k, $k = 0, 1, 2, \ldots$.