Problem 11. Prove that

$$\sin(x - y)\sin(x + y) = (\sin x - \sin y)(\sin x + \sin y)$$

for all real x and y.

Solution 11. By the angle addition formula for the sine,

$$\begin{aligned}
\sin(x - y)\sin(x + y) &= (\sin x \cos y - \cos x \sin y)(\sin x \cos y + \cos x \sin y) \\
&= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\
&= \sin^2 x(1 - \sin^2 y) - (1 - \sin^2 x)\sin^2 y \\
&= \sin^2 x - \sin^2 y \\
&= (\sin x - \sin y)(\sin x + \sin y).
\end{aligned}$$