

Problem 1. Moe, whose favorite color is red, has a large drawer filled with single red socks. The socks are in 21 different shades of red from very very light red to very very dark red. On a nearby wall Moe has a color chart showing the 21 shades, in order from lightest to darkest. Moe decides he can wear a pair of sock if the socks in the pair are of the same shade or if the two socks have shades that are adjacent shades on his chart. One dark morning when the power is out, Moe, who cannot see the socks, pulls several socks from his drawer, planning to examine them in the light from a window. Moe needs *two* wearable pairs of socks to get through the day. What is the minimal number of socks that Moe must choose to be certain that he has *two* wearable pairs among the selected socks? You may assume that there are several socks of each shade in the drawer. Solutions must have the correct answer and proof that the answer is indeed the minimum.

Solution 1. Moe must take at least fourteen socks to guarantee that he has at least two wearable pair.

First suppose that Moe wants just one wearable pair. Then he must select at least 12 socks. To prove this, assign each sock shade a number between 1 and 21, where 1 denotes the lightest shade and 21 denotes the darkest shade. If Moe selects eleven socks of shades 1, 3, 5, ..., 21, then there is no wearable pair. Thus eleven socks does not guarantee a wearable pair. However, with twelve socks, there must be a wearable pair. If not, then no two of the socks are the same shade and no two socks have consecutive shade numbers. In this case the socks would have shades

$$1 \leq n_1 \leq n_2 - 2 \leq n_4 - 4 \leq \cdots \leq n_{12} - 22,$$

so $n_{12} \geq 23$, However, this is impossible because there are only 21

different shades. Thus if Moe selects 12 socks, he is guaranteed at least one wearable pair.

Therefore, if Moe selects fourteen socks, then he can find a wearable pair. Once this pair is removed there are 12 socks remaining, so the remaining set has at least one additional wearable pair. Finally, if Moe chooses thirteen socks with three of shade 1 and one of each of shade 3, 5, \dots , 21, then there is a pair of shade 1, but once these are removed there are eleven socks left but no wearable pair.