Solid of Revolution: The Washer Method

0 seconds ago by admin

Recall that we have a solid of revolution defined by rotating about the x-axis the region where

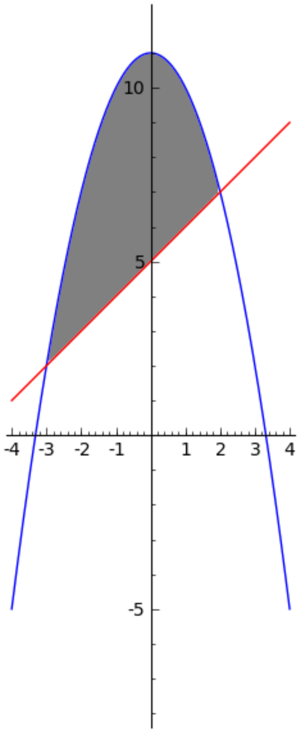
$$x + 5 \leq y \leq 11 - x^2$$

```
x,y = var('x','y')

R = 11 - x^2
r = x + 5

R,r
```

$(-x^2 + 11, x + 5)$

We can plot this xy-region to see what it looks like.

```
P = plot(R(x=x), (x,-4,4), color="blue") + plot(r(x=x), (x,-4,4), color="red") + region_plot( [R(x=x) >= y, r(x=x) <= y], (-4,4), (-8,12), incol="gray")


P.set_aspect_ratio(1.0)
P
```



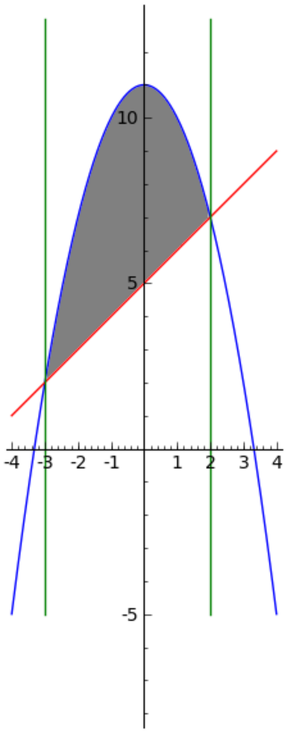
An important step to do is to compute the bounds here. This can be done by setting $R(x) = r(x)$ and solving for x.

```
sol = solve(R(x=x) == r(x=x), x)
a = sol[0].right_hand_side()
b = sol[1].right_hand_side()
sol
```

$[x = (-3), x = 2]$

Let's draw these boundary lines on the plot.

```
Q = P + line([(sol[0].right_hand_side(),-5), (sol[0].right_hand_side(), 13)], color="green") +
line([(sol[1].right_hand_side(),-5), (sol[1].right_hand_side(), 13)], color="green")
Q
```



We can now compute our integral using the washer method.

$$
\begin{aligned}
V &= \int_{-3}^{2} \pi \left[(11 - x^2)^2 - (x+5)^2 \right] dx \\
&= \pi \int_{-3}^{2} \left[(121 - 22x^2 + x^4) - (x^2 + 10x + 25) \right] dx \\
&= \pi \int_{-3}^{2} \left[x^4 - 23x^2 - 10x + 96 \right] dx \\
&= \pi \left[\tfrac{1}{5} x^5 - \tfrac{23}{3} x^3 - 5x^2 + 96x \right]_{-3}^{2} \\
&= \pi \left[\left(\tfrac{1}{5}(2)^5 - \tfrac{23}{3}(2)^3 - 5(2)^2 + 96(2) \right) - \left(\tfrac{1}{5}(-3)^5 - \tfrac{23}{3}(-3)^3 - 5(-3)^2 + 96(-3) \right) \right] \\
&\vdots \quad \text{(simplifying)} \\
&= \tfrac{875}{3} \pi.
\end{aligned}
$$

```
from sage.symbolic.integration.integral import definite_integral

V = pi * definite_integral( (11-x^2)^2 - (x+5)^2, x,  a, b)

V
```

$\frac{875}{3} \pi$