Learning Objectives for Math 165

Chapter 2 — Limits

Section 2.1: Average Rate of Change.

- State the definition of average rate of change
- Describe what the rate of change does and does not tell us in a given context
- Interpet and work with analytic, graphical, and numerical information
- Work successfully and efficiently with function notation, substitution in functions ($x \to a + h$), how a graph represents a function
- Describe the relationship between the secant line and the average rate of change and apply the relationship in a given context
- Describe the relationship between the tangent line and the instantaneous rate of change and apply the relationship in a given context.

Section 2.2: Working with limits, and Section 2.4

- State and work with the definition of limit as described on page 66
- Demonstrate, describe, and recognize ways in which limits do not exist
- Evaluate limits given analytic, graphical, numerical function information
- Explain and give examples illustrating the indeterminent nature of $\frac{0}{0}$ forms
- Describe in simple language the statements of limit laws and use these laws to evaluate limits.
- Evaluate one sided limits and describe relationship between limits and one-sided limits.
- Justify (graphically, numerically) the approximation $\sin x \approx x$ for small x and demonstrate that the approximation may not be good for large x.
- Work with limit statements to obtain other approximations in a manner similar to the $\sin x \approx x$ derivation

Section 2.5: Continuity.

- State the definition of continuity and use the definition to ascertain the continuity or non continuity of a function at a point
- Explain and illustrate ways in which function can be discontinuous.
- Recognize graphs of continuous functions and recognize graphs of discontinuous functions
- Generate functions and graphs of functions demonstrating the different types of discontinuities
- Make a continuous extension of a function.

Section 2.6: Infinite limits and limits at Infinity

- Work with $\frac{1}{0}$ forms in the context of one-sided limits to determine existence and location of vertical asymptotes
- Use limits to determine existence and location of horizontal asymptotes
- State the definitions of vertical and horizontal asymptotes (pgs. 105, 111) and work with these definitions to create graphs of functions.
- Evaluate and work with limits $x \to \pm\infty$ by working with dominant terms.
- Explain the difference between $x \to \infty$ and $x = \infty$.

Chapter 3 — Differentiation

Section 3.1: Tangents and the Derivative at a Point.

- State the limit definition of derivative of a function $f(x)$ at a point $(x, f(x)$ and use the limit definition to calculate a derivative or identify where the derivative fails to exist at a point.
- Interpret the limit definition of the derivative of a function $f(x)$ at a point $x = a$ as
  - the slope of the graph of a function $f(x)$ at a point $x = a$,
  - the slope of the tangent to the curve at a point $x = a$, and
  - the rate of change of a function $f(x)$ with respect to x at $x = a$.
- Recognize and be flexible in the use of different representations of the definition of derivative, including

$$f'(a) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

- Interpret and be able to represent the definition of derivative analytically, graphically, and numerically.
- Given a context, apply units to the derivative
- Given the derivative of a function at a point, sketch a graph of the “elements” (i.e., local linearity).

Section 3.2: The Derivative as a Function

- Interpret the derivative of a function and state the domain for the derivative, namely, identifying the points of a function f for which the derivative does not exist.
- Analytically determine the derivative of a function with respect to x at a generic x-value.

- Interpret and use (apply?) different notation representing the derivative of a function including

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = D_x f(x)$$

- Estimate the derivative of a function f with respect to x at a point $(a, f(a)$ using the graph of the function and the tangent line to the function at $x = a$. Similarly, graph the function and the tangent line to the function for $x = a$ when given $f'(a)$.
- Give examples of functions where the derivative at a point does not exist and why.
- Give examples of continuous functions which are not differentiable.

Section 3.3: Differentiation Rules.

- Calculate derivatives of functions (without the use of technology) by applying differentiation rules, including the derivative of:
  - a constant function, a constant multiple of a function, the power of x, a sum of functions, a product of functions, a quotient of functions, and exponential functions.
- Verify analytically (i.e., using the limit definition of derivative) the derivative rules for a constant function, a constant multiple of a function, the power of x (where n is an integer or $n = \pm 1/2$, a sum of functions, and exponential functions (which explains why we use e).
- Interpret and make use of notation for second- and higher-order derivatives as well as be able to calculate higher-order derivatives.

Section 3.4: The Derivative as a Rate of Change

- Interpret the difference quotient as an average rate of change over a specified interval. Interpret the instantaneous rate of change as the rate at which the function is changing at a point. Attach units to the difference quotient and the instantaneous rate of change based on the context.
- Given a position function, solve for the (instantaneous) velocity of the object using the definition of derivative of the object's position with respect to time. Solve for the speed of an object or graph the speed as a function of time when speed = $|v'(t)| = |\frac{ds}{dt}|$
- Relate the acceleration of an object to the object's position and object's velocity, (i.e., Acceleration = $\frac{d^2s}{dt^2} = s''(t) = \frac{dv}{dt} = v'(t)$). Assign units to acceleration based on the context.
- Apply knowledge of the position, velocity, and acceleration functions to solve problems related to motion.

Section 3.5: Derivatives of Trigonometric Functions

- Use the limit definition to express the derivatives of the trigonometric functions.
- Graphically demonstrate why $\frac{d}{dx}(\sin(x)) = \cos(x)$ and why $\frac{d}{dx}(\cos(x)) = -\sin(x)$.
- Use the derivative rules to establish the derivative rules for $\tan(x)$, $\sec(x)$, $\cot(x)$, and $\csc(x)$.
- Use the derivative rules to evaluate the derivatives of functions which contain the trigonometric functions.

Section 3.6: The Chain Rule

- Apply the Chain Rule to find the derivative of a composition of functions. Identify the order at which functions are embedded in another, particularly when the chain rule needs to be repeated to find the derivative of a function.

Section 3.7: Implicit DIfferentiation

- Apply the chain rule to differentiate implicitly defined functions
- Find higher-order derivatives using implict differentiation

Section 3.8: Derivatives of Inverse Functions and Logarithms

- State the definition one-to-one function (Section 1.6). State why only a one-to-one function can have an inverse.
- Produce a formula for the inverse of a one-to-one function given a formula for the function and perform the algebra (function composition) to demonstrate that you have the correct inverse.
- Given the graph of a one-to-one function, draw the graph of the inverse function.
- Intepreting the derivative of a function as the slope of a tangent line, draw a picture illustrating that $\left.\frac{d(f^{-1})}{dx}\right|_{x=b} = 1/f'(f^{-1}(b))$.
- State the definition of the logarithm (Section 1.6). Demonstrate the ability to manipulate logarithms (and exponentials)
- State and work with the derivative of the natural logarithm function.
- Derive the definition of the logarithm by differentiating the expression $e^{\ln x} = x$.
- Explain the reasoning leading to $\frac{d}{dx}\ln|x| = \frac{1}{x}$.

Section 3.9: Inverse Trigonometric Functions

- Define the inverse sine and cosine functions, including the domain, and range of these functions and their relation to the sine and cosine function. Draw the graphs of the inverse sine and cosine functions, and describe the relation to the graphs of the sine and cosine functions.

- Define the inverse tangent function, including the domain, and range of the function and its relation to the tangent function. Draw the graphs of the inverse tangent function, and describe the relation to the graph of the tangent functions.
- Explain the sin^{-1} notation and how it differs from $(\sin x)^{-1}$.
- Be able to write down the derivatives of the inverse sine, cosine, and tangent functions. Be able to apply these derivatives as needed in the contest of the project, quotient, and chain rules.
- Be able to derive the derivative of $\tan^{-1} x$ and $\sin^{-1} x$ by applying the chain rule to the equations $tan(\tan^{-1} x) = x$ and $\sin(\sin^{-1} x)$.

Section 3.10: Related Rates

- Set up and solve related rates problems, clearly documenting work as elucidated in the box on Page 194.
- At the conclusion of a related rates problem write a sentence declaring exactly what was found and what it means.
-

Section 3.11: Linearization and Differentials

- Given a function that is differentiable at a point $(c, f(c))$ be able do describe what you will see if you zoom in on the graph at this point.
- Be able to define and work with the linearization of a function at a point, and describe the relationship between the linearization and the tangent line.
- Use the linearization to calculate an approximation to a function near a "nice point."
- Be able to define and work with the differential. Explain the relationship between the quantities Δf and df and be able to draw and clearly label an picture to illustrate these quantities.
- Calculate a differential and use it to estimate the change in a function over a small interval.

Chapter 4 — Applications of Derivatives

Section 4.1: Extreme Values of Functions.

- Write and explain the definition of absolute maximum and absolute minimum of a function.
- Write the Extreme Value Theorem. Give examples of (i) function(s) that are continuous on an interval (a, b) but do not have an absolute maximum or an absolute minimum; (ii) function(s) that are not continuous on an interval $[a, b]$ and do not have an absolute maximum or absolute minimum.

- Explain the difference between an absolute extreme point and a relative extreme point.
- Given a function f continuous on a closed interval $[a, b]$, find the values at which f takes on its absolute maximum and minimum values and find the values of extreme values.
- Write the definition of critical points and explain their importance in finding relative and absolute extreme points.
- Identify both the candidates for extreme values of a function and the extreme values of the functions, if relalvant.

Section 4.3: Monotonic Functions and the First Derivative Test

- Explain what it means for a function to be increasing (decreasing) on an interval. Give graphical and symbolic interpretations.
- Given a function f, use the first derivative to identify the intervals on which f is increasing, decreasing.
- Use the first derivative test to determine the nature (relative maximum, relative minimum, neither) of a critical point.
- Justify the First Derivative Test as stated on Page 240. Use graphs and knowledge about monotonicity in your justification.

Section 4.4: Concavity and Curve Sketching

- State the definitions of concave up and concave down.
- Use slopes of tangents to a graph to relate the concavity of a function to the increasing/decreasing nature of the first derivative.
- Explain what it means for a point to be an inflection point and use analysis of the second derivative to identify inflection points.
- Use the second derivative test to classify critical points.
- Give examples to show that the second derivative test yields no information for a critical point c if $f''(c) = 0$.
- Use the tools of calculus and algebra to sketch, by hand, good graphs of functions, including labels of the intercepts, critical points, inflection points, asymptotes, intervals of monotonicity, and documentation of concavity.
- Given graphical and textual information about a function f (e.g. intercepts, where f' is $\pm$ or a graph of f', etc.) draw a graph of $y = f(x)$.
- Given a graph of a function f, sketch a graph of f'. Given a graph of f', sketch a graph of f.

Section 4.5: Indeterminant Forms and L'Hôpital's Rule

- Explain why $\frac{0}{0}$ and $\frac{\infty}{\infty}$ are indeterminant forms. Explain what is meant by indeterminant form.
- Use L'Hôpital's rule to evaluate limits involving indeterminant forms.
- Use logarithms and L'Hôpital's rule to evaluate limits of indeterminant forms such as 0^0, ∞^0, 1^∞.

Section 4.6: Applied Optimization

- Use the tools of calculus to solve applied optimization problems. Write complete solutions including illustrations, identification of variables, verification that the solution produced is the extreme sought.

Section 4.7: Newton's Method

- Use Newton's method to find approximate solutions to equations such as $f(x) = 0$ or $g(x) = h(x)$.
- Justify and derive the Newton's method recursion as on Page 275.

Section 4.8: Antiderivatives.

- Evaluate antiderivatives of elementary functions.
- Explain the significance of the $+C$ for antiderivatives.
- Find the value of the constant $+C$ in the context of initial value problems.

Chapter 5— Integration

Section 5.1: Area and Estimating with Finite Sums.

- Describe upper and lower sums and what they tell you about area.
- Given a partition and set $\{t_1, \dots, t_n\}$ of points, with a single t_i in each subinterval, draw the rectangles and calculate the associated Riemann Sum.
- Use Riemann sums to approximate distance and net distance traveled, given a velocity function. During this process keep track of units and know which units go with each quantity.
- Approximate the average of a function and explain why this approximation is also a Riemann sum.

Section 5.2: Sigma Notation and Limits of Finite Sums

- Given a sum written in sigma notation, write out the expanded from of the sum, and evaluate the sum.
- Given a sum displayed expanded form write the sum using sigma notation.
- Be able to manipulate sums using the algebra rules on page 308 of the text book.

- Provide all details in using an equal interval, $n+1$ point partition, with right or left endpoints to obtain a Riemann sum. Given the formulas on Page 309, evaluate the limit of the result as $n \to \infty$ and interpret the result.
- Explain the contributions given by positive and negative terms of a Riemann sum and give a geometric interpretation.

Section 5.3: The Definite Integral.

- Write the limit definition of definite integral (Page 315), including the relationship to a partition of an interval and explanations of all notation.
- Use the rules in Table 5.4, Page 317, to evaluate and manipulate definite integrals.
- Explain and describe the relationship of the definite integral to area under the graph of a non-negative function.
- Find the average value of a continuous function over an interval $*$ and explain how the average value formula arises from the definition of the definite integral.

Section 5.4: The Fundamental Theorem of Calculus.

- Write the statement of the Fundamental Theorem of Calculus (Part 1) and explain what the theorem says about definite integrals.
- Provide an outline of the proof of the FTC with diagrams. See Page 326.
- Write the statement of the Fundamental Theorem of Calculus (Part 2) and explain what the theorem says about definite integrals.
- Use the FTC (Parts 1 and 2) to evaluate derivatives of functions defined by integrals and to evaluate definite integrals. (say something about the use of the chain rule?)
- If $f(t)$ is a rate function, use Riemann sums to explain the meaning of $\int_a^b f(t)\,dt$ as a net change.

Section 5.5: Definite Integration and the Substitution Method.

- Evaluate indefinite integrals using the substitution method (when needed) showing complete work with differentials.
- Given an integral, identify and implement the appropriate substitution.
- Find the antiderivatives of $\sin^2\theta$ and $\cos^2\theta$.

Section 5.6: Substitution and Area Between Curves

- Explain, in terms of area, the difference between $\int_a^b f(x)\,dx$ and $\int_a^b |f(x)|\,dx$, with reasons. Evaluate each of these integrals for a given function f.
- Given a velocity function $v(t)$, explain the difference between $\int_a^b v(t)\,dt$ and $\int_a^b |v(t)|\,dt$, with reasons.
- Evaluate definite integrals and use substitution when needed. Make the appropriate changes in the limits of integration when performing such an integration. $*$ I changed the wording. $*$
- Given two functions $y = f(x)$ and $y = g(x)$, be able to set up and evaluate the integral(s) to calculate the area between the graphs of the two curves over a given interval. $*$ I changed the wording. $*$

Chapter 7 — Integrals and Transcendental Functions

Section 7.2: Exponential Change and Separable Differential Equations.

- Solve simple separable equations (with and without initial conditions.)
- After solving a differential equation, be able to answer further questions about the situation being modeled.
- State the definitions of half-life and doubling-time and work with these ideas.

- Given a written description of a change phenomenon, write a separable differential equation to model the phenomenon.
- Given a separable differential equation and a context, write a short paragraph describing what the equation is modeling.
- Give examples of phenomena for which the rate of change of a quantity is proportional to the value of the quantity and show that the quantity is modeled by an exponential function.