1) Let \( a_n = \frac{n+2}{n} \). Prove directly from the definition of convergence of a sequence that \( a_n \to 1 \).

2) Let \( f : \mathbb{R} \setminus \{1\} \to \mathbb{R} \) by \( f(x) = \frac{x}{x-1} \). Determine whether \( f \) is one-to-one and/or onto.

3) Prove that the sequence \( \{a_n\} \) has a convergent subsequence where

\[
a_n = \frac{7n \sin n + 3 \cos^3 n^2}{n(1 + (-1)^n) + 1}.
\]

4) Let \( E = \{1/m + 1/n : n, m \in \mathbb{N}\} \).
   a) Find \( \inf(E) \) (and justify your answer)
   b) Prove that for any \( n \in \mathbb{N} \) that \( 1/n \) is a limit point of \( E \).

5) Suppose \( \{a_k\} \) is a sequence of positive real numbers for which \( a_{k+1}/a_k \leq b \) for some \( b < 1 \). Prove that \( \sum a_k \) converges by showing that the sequence \( \{s_n\} \) of partial sums is contractive.

6) Suppose \( \{a_k\} \) is convergent and \( \{b_k\} \) diverges to \( +\infty \). Prove that \( \{a_k + b_k\} \) diverges to \( +\infty \).

   Here’s a start: Let \( M > 0 \). We (i.e. you) need to find positive integer \( N \) such that for \( n \geq N \), \( a_k + b_k \geq M \). First work on \( \{a_k\} \). For some \( B \geq 0 \) we have \( |a_k| \leq B \) for all \( k \).
(Why?) You finish the rest.