Instructions: Read the questions carefully and answer all parts of the questions.

1) (5 pt) Write out Taylor’s Formula (with the remainder) for \( f(x) = 1/x \) centered at \( a = 1 \) and \( n = 3 \). (So that the Taylor polynomial is third order, the remainder term is 4th order.)

2) (10 pt) Suppose that \( f \) is twice continuously differentiable on \( I = [-1, 1] \), \( f(0) = 0 \), \( f'(0) = 0 \) and \( f'' \geq 0 \) on \( I \). Prove that \( f(x) \geq 0 \) on \( I \). (Suggestion: prove by contradiction, using the MVT.)

3) (10 pt) State and prove Rolle’s Theorem.

4) (10 pt) Prove that \( f(x) = x^{1/2} \) is uniformly continuous on \((0, \infty)\).

5) (10 pt) Prove the product rule: If \( f \) and \( g \) are differentiable at \( x = a \) then \( fg \) is differentiable at \( a \) and \( (fg)'(a) = f'(a)g(a) + f(a)g'(a) \).

6) Either prove or give a counterexample:
   a) If \( f \) is bounded then \( f \) is continuous.
   b) If \( f \) is continuous then \( f \) is differentiable.
   c) If \( |f| \) is differentiable, then \( f' \) is continuous.

7) (5pt) Let \( f(x) = x \sin(1/x) \) for \( x \neq 0 \) and define \( f(0) = 1 \). Prove \( f \) is discontinuous at 0.

8) (6pt) Let \( f(x) = x^3 - x^2 \sin x \).
   a) Prove (using the appropriate definition) that \( \lim_{x \to \infty} f(x) = \infty \).
   b) Prove that \( f : \mathbb{R} \to \mathbb{R} \) is surjective. (You can assume that \( \lim_{x \to -\infty} f(x) = -\infty \).)

9) (6pts) Let \( f(x) = x \cos(x) - x \).
   a) Use the Mean Value Theorem to prove \( f \) is strictly monotone decreasing on \([-\pi, \pi]\).
   b) Prove \( f^{-1} \) is continuous on \([0, 2\pi]\), differentiable on \((0, 2)\) and find a formula for the derivative of \( f^{-1}(y) \) for \( y \in (0, 2\pi) \).

10) (6pts) Let \( f(x) = x^{1/3} \). Prove \( f \) is uniformly continuous on \( E = (0, 100) \), but does not satisfy the Lipschitz condition on \( E \).