Vertex identifying codes in infinite grids

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Discrete Geometry and Combinatorics Seminar
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Joint Work

This talk is based mostly on work by and with:

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Failed node

We begin with a graph knowing that, at some point, there will be a node that fails.
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I.e., a computer downloads a virus.
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The SysAdmin wants to discover who was the originator of the failure but doesn’t want to monitor all nodes.
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I.e., a computer downloads a virus.

Once it fails, it spreads the failure to its neighbors.

The SysAdmin wants to discover who was the originator of the failure but doesn’t want to monitor all nodes.

So he monitors a small subset, called a code.
An example

Consider the following code on this familiar graph.

The code vertices are in red.
An example

Consider the following code on this familiar graph.

This code will detect when any failure occurs.
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This code will distinguish which node fails.
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*If the magenta vertex reads failure and the red do not, then the failed node must be the green one.*
An example

Consider the following code on this familiar graph.

This code will distinguish which node fails.

*If the magenta vertices read failure, then the failed node must be the green/magenta one.*
Identifying code

Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. 

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Let $N[v] = N(v) \cup \{v\}$ be the closed neighborhood of a vertex.
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$$N[v] \cap C \neq \emptyset \quad \forall v \in V(G).$$
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**Definition**

An **identifying code** is a $C \subseteq V(G)$ such that:

- $C$ is a dominating set and
- $N[v] \cap C \neq N[w] \cap C$ for all $v \neq w$. 


Let's return to the example of the Petersen graph.
Petersen Code

Let’s return to the example of the Petersen graph.

The red vertices are an identifying code of size 5.
Petersen Code

Let’s return to the example of the Petersen graph.

This is a smaller set.
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This is a smaller set.

Smaller is better for this problem. But, is it a code?
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Yes!
Petersen Code

Let’s return to the example of the Petersen graph.

This is a smaller set.

Is there a smaller code?
Lower bound

If $C$ is a code of size 3 in the Petersen graph, $P_{10}$,
Let $G$ be a graph with identifying code $C$, then $|C| \geq \lceil \log_2 (n+1) \rceil$.

Proof. If $C$ is a code of size 3 in the Petersen graph, $P_{10}$, then the map $f : V(P_{10}) \to 2C \setminus \emptyset$ is an injection. Thus, $10 \leq 2^3 - 1$, a contradiction. □
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f(v) = N[v] \cap C
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Theorem (Karpovsky, Chakrabarty, Levitin 1998)

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If $C$ is a code, then the map

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The complete graph has no code.
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The complete graph, $K_n$, $n \geq 1$, has no code.

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This is because

$$N[v] \cap C = N[w] \cap C,$$

for all $C \subseteq V(K_n)$,

and all $v, w \in V(G)$. 
If there exists a code in graph $G$, . . .
Codes among us

If there exists a code in graph $G$,

then $N[v] \neq N[w]$, for all distinct $v, w \in V(G)$. 

Theorem (Karpovsky, Chakrabarty, Levitin 1998)

A graph $G$ admits an identifying code if and only if $N[v] \neq N[w]$, for all distinct $v, w \in V(G)$.

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(Closed neighborhoods are always nonempty.)
Theorem (Karpovsky, Chakrabarty, Levitin 1998)

A graph $G$ admits an identifying code if and only if $N[v] \neq N[w]$, for all distinct $v, w \in V(G)$.

Proof.

If there exists a code in graph $G$,

then $N[v] \neq N[w]$, for all distinct $v, w \in V(G)$.

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Existence

It is natural to ask about codes in the random graph $G_{n,p}$. 
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Theorem (Frieze, M., Moncel, Ruszinkó, Smyth 2007)

If $\mathcal{E}(n, p)$ is the event that $G_{n,p}$ has an identifying code, then

$$\lim_{n \to \infty} \Pr(\mathcal{E}(n, p))$$

is

$$1, \quad \text{if } p = o(n^{-2});$$
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1, \quad \text{if } p = o(n^{-2});

$$e^{-c_1/2}, \quad \text{if } p = c_1 n^{-2};$$

0, \quad \text{if } p = 1 - \ln n + \omega(1) + \ln \ln n.$$
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Theorem (Frieze, M., Moncel, Ruszinkó, Smyth 2007)

If $E(n, p)$ is the event that $G_{n,p}$ has an identifying code, then

$$\lim_{n \to \infty} \Pr (E(n, p))$$

is

$$
\begin{align*}
1, & \quad \text{if } p = o(n^{-2}); \\
e^{-c_1/2}, & \quad \text{if } p = c_1 n^{-2}; \\
0, & \quad \text{if } \omega(n^{-2}) = p = \frac{\ln n + \ln \ln n - \omega(1)}{2n};
\end{align*}
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- $1$, if $p = o(n^{-2})$;
- $e^{-c_1/2}$, if $p = c_1 n^{-2}$;
- $0$, if $\omega(n^{-2}) = p = \frac{\ln n + \ln \ln n - \omega(1)}{2n}$;
- $e^{-e^{-c_2/4}}$, if $p = \frac{\ln n + \ln \ln n + c_2}{2n}$;
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e^{-e^{-c_2/4}}, \quad \text{if } p = \frac{\ln n + \ln \ln n + c_2}{2n}; \\
1, \quad \text{if } \frac{\ln n + \ln \ln n + \omega(1)}{2n} = p \\
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- $1$, if $p = o(n^{-2})$;
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- $0$, if $\omega(n^{-2}) = p = \frac{\ln n + \ln \ln n - \omega(1)}{2n}$;
- $e^{-e^{-c_2/4}}$, if $p = \frac{\ln n + \ln \ln n + c_2}{2n}$;
- $1$, if $\frac{\ln n + \ln \ln n + \omega(1)}{2n} = p$
  and $p = 1 - \frac{\ln n + \omega(1)}{n}$;
- $e^{-e^{-c_3}} (1 + e^{-c_3})$, if $p = 1 - \frac{\ln n + c_3}{n}$. 
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\end{align*}$$
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Code sizes in random graphs

For a graph $G$, define $c(G)$
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$$c(G_{n,p}) \sim \frac{2 \log n}{\log(1/q)}.$$
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**Theorem (Frieze, M., Moncel, Ruszinkó, Smyth 2007)**

Let $p, 1 - p \geq 4 \ln \ln n / \ln n$,

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The proof uses an inequality of Suen. Suen's inequality resembles the Lovász Local Lemma, in that there is a dependency graph.
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Size versus density

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Density will be intuitive for all of our examples, so we will not worry about potential issues involved.
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Theorem (Karpovsky, Chakrabarty, Levitin 1998)

If $C$ is a vertex identifying code on a $d$-regular graph $G$ with $|V(G)| = N$, then

$$|C| \geq \frac{2N}{d + 2}.$$
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Theorem (Karpovsky, Chakrabarty, Levitin 1998)
If $C$ is a vertex identifying code of density $D$ on a $d$-regular graph $G$, then

$$D \geq \frac{2}{d + 2}.$$
Triangular grid

Lower Bound: $d$-regular, $N$ vertices: For every code, $C$, $|C| \geq 2N_d + 2$. Take the limit.

Upper Bound: The following construction.
Let $G$ be a grid and $D(G)$ denote the infimum of the density of a code in $G$. 

**Triangular grid**

![Triangular grid diagram]
Triangular grid

\[ D(\mathbb{T}) = \frac{1}{4} \quad [\text{KCL}] \]
Triangular grid

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- **Lower Bound**: \( d \)-regular, \( N \) vertices: For every code, \( C \),
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- **Lower Bound**: \( d \)-regular, \( N \) vertices: For every code, \( C \),
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- **Lower Bound**: \( d \)-regular, \( N \) vertices: For every code, \( C \),
  \[ D \geq \frac{2}{d + 2}. \quad \text{Take the limit.} \]
Triangular grid

\[ D(\mathbb{T}) = \frac{1}{4} \]  \[ \text{[KCL]} \]

- **Lower Bound**: \( d \)-regular, \( N \) vertices: For every code, \( C \),
  \[ D \geq \frac{2}{6 + 2}. \]  Take the limit.
Triangular grid

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- **Lower Bound:** \(d\)-regular, \(N\) vertices: For every code, \(C\),
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- **Upper Bound:** The following construction.
Triangular grid

\[ D(\mathbb{T}) = \frac{1}{4} \]  

[\text{KCL}]

- **Lower Bound**: \( d \)-regular, \( N \) vertices: For every code, \( C \),

\[
D \geq \frac{2}{6+2}. \quad \text{Take the limit.}
\]

- **Upper Bound**: The following construction.
Square grid

\[ D(\mathbb{Z}^2) = 20 \]


Square grid

\[ D(\mathbb{Z}^2) = \frac{7}{20} \]
Square grid

\[ D(\mathbb{Z}^2) = \frac{7}{20} \]

- **Lower Bound:** Litsyn, Merksamer 2004.
Square grid

\[ D(\mathbb{Z}^2) = \frac{7}{20} \]

- **Lower Bound**: Litsyn, Merksamer 2004.
King grid

\[ D(K) = 2^9 \]


King grid

\[ D(\mathbb{K}) = \frac{2}{9} \]
King grid

\[ D(\mathcal{K}) = \frac{2}{9} \]

King grid

$$D(\mathbb{K}) = \frac{2}{9}$$

Hexagonal grid
\[
\frac{12}{29} \leq D(\mathbb{H}) \leq \]

Hexagonal grid
Hexagonal grid

\[ \frac{12}{29} \leq D(\mathcal{H}) \leq \]

Lower Bound: Cranston, Yu 2009.

Hexagonal grid

\[
\frac{12}{29} \leq D(\mathbb{H}) \leq
\]

- **Lower Bound:** Cranston, Yu 2009.
Hexagonal grid

\[ \frac{12}{29} \leq D(\mathbb{H}) \leq \frac{3}{7} \]

- **Lower Bound:** Cranston, Yu 2009.
- **Upper Bound:** Cohen, Honkala, Lobstein, Zémor 2000.
0.4138 \leq D(\mathbb{H}) \leq \frac{3}{7} \cdot 0.4286

- **Lower Bound**: Cranston, Yu 2009.
Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$.  

**r-identifying code**

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**r-identifying code**
$r$-identifying code

Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$.

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**Definition**

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**Definition**

An **$r$-identifying code** is a $C \subseteq V(G)$ such that:

- $C$ is a dominating set and
- $N_r[v] \cap C \neq N_r[w] \cap C$ for all $v \neq w$. 
### Hexagonal grid, large $r$

<table>
<thead>
<tr>
<th>$r$</th>
<th>Lower Bounds</th>
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</tr>
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<td>0.0187</td>
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</table>

\[
\begin{align*}
\frac{2}{5r+3}, & \quad \text{if } r \text{ is even}; \\
\frac{2}{5r+2}, & \quad \text{if } r \text{ is odd.}
\end{align*}
\]

\[
\left\{ \leq D_r(\mathbb{H}) \sim \frac{8}{9r} \right. 
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- **Upper Bound**: Honkala, Laihonen 2003.
Hexagonal grid, large $r$

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\frac{2}{5r+3}, \quad \text{if } r \text{ is even}; \quad \frac{2}{5r+2}, \quad \text{if } r \text{ is odd.}
\]

\[
\left\{ \frac{5r+3}{6r(r+1)}, \quad \text{if } r \text{ is even}; \quad \frac{5r^2+10r-3}{(6r-2)(r+1)^2}, \quad \text{if } r \text{ is odd.} \right\}
\]

- **Lower Bound:** Cohen, Honkala, Hudry, Lobstein 2001.
- **New Upper Bound:** Stanton 2010+.
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\left\{ \frac{2}{5r+3}, \quad \text{if } r \text{ is even; } \right. \\
\left. \frac{2}{5r+2}, \quad \text{if } r \text{ is odd. } \right\} \leq D_r(\mathbb{H}) \lesssim \frac{5}{6r}
\]

- **Lower Bound:** Cohen, Honkala, Hudry, Lobstein 2001.
- **New Upper Bound:** Stanton 2010+.
Hexagonal grid, large $r$

$r = 6$

\[
\begin{align*}
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\end{align*}
\]

\[\leq D_r(\mathbb{H}) \lesssim \frac{5}{6r}\]

- **Lower Bound:** Cohen, Honkala, Hudry, Lobstein 2001.
- **New Upper Bound:** Stanton 2010+.
Hexagonal grid, large $r$

$$r = 7$$

$$\frac{2}{5r+3}, \quad \text{if } r \text{ is even;}$$

$$\frac{2}{5r+2}, \quad \text{if } r \text{ is odd.}$$

$$\left\{ \frac{2}{5r+3}, \frac{2}{5r+2} \right\} \leq D_r(\mathbb{H}) \lesssim \frac{5}{6r}$$

- **New Upper Bound**: Stanton 2010+.
Square grid, $r = 2$

\[ \frac{3}{20} \leq D_2(\mathbb{H}) \leq \frac{5}{29} \]
Square grid, $r = 2$

\[
\frac{3}{20} \leq D_2(\mathbb{H}) \leq \frac{5}{29}
\]

- **Lower Bound:** Charon, Honkala, Hudry, Lobstein 2001.
Square grid, \( r = 2 \)

\[
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- **Lower Bound:** Charon, Honkala, Hudry, Lobstein 2001.
- **Upper Bound:** Honkala, Lobstein 2002.
Square grid, $r = 2$

\[
\frac{6}{37} \leq D_2(\mathbb{H}) \leq \frac{5}{29}
\]

- **New Lower Bound:** M., Stanton 2010.
- **Upper Bound:** Honkala, Lobstein 2002.
Square grid, $r = 2$

\[ 0.1622 \leq D_2(\mathbb{H}) \leq 0.1724 \]

- **New Lower Bound**: M., Stanton 2010.
- **Upper Bound**: Honkala, Lobstein 2002.
Hexagonal grid, $r = 2$

\[ \frac{2}{11} \leq D_2(\mathbb{H}) \leq \frac{4}{19} \]

- **Lower Bound:** Karpovsky, Chakrabarty, Levitin 1998.
Hexagonal grid, $r = 2$

$$\frac{2}{11} \leq D_2(\mathbb{H}) \leq \frac{4}{19}$$

- **Lower Bound**: Karpovsky, Chakrabarty, Levitin 1998.
Hexagonal grid, $r = 2$

\[ \frac{1}{5} \leq D_2(\mathbb{H}) \leq \frac{4}{19} \]

- **New Lower Bound:** M., Stanton 2010.
- **Upper Bound:** Charon, Hudry, Lobstein 2004.
Hexagonal grid, $r = 2$

$0.2 \leq D_2(\mathbb{H}) \leq 0.2105$

- **New Lower Bound:** M., Stanton 2010.
- **Upper Bound:** Charon, Hudry, Lobstein 2004.
Hexagonal grid, $r = 3$

\[
\frac{2}{17} \leq D_2(\mathbb{H}) \leq \frac{1}{6}
\]
Hexagonal grid, $r = 3$

\[ \frac{2}{17} \leq D_2(\mathbb{H}) \leq \frac{1}{6} \]

- **Lower Bound:** Charon, Honkala, Hudry, Lobstein 2001.
Hexagonal grid, $r = 3$

\[ \frac{2}{17} \leq D_2(\mathbb{H}) \leq \frac{1}{6} \]

Hexagonal grid, $r = 3$

$$\frac{3}{25} \leq D_2(\mathbb{H}) \leq \frac{1}{6}$$

- **NEW Lower Bound:** Stanton 2010++.
- **Upper Bound:** Charon, Hudry, Lobstein 2004.
Hexagonal grid, $r = 3$

$0.12 \leq D_2(\mathbb{H}) \leq 0.1667$

- **NEW Lower Bound:** Stanton 2010++.
- **Upper Bound:** Charon, Hudry, Lobstein 2004.
Hex grid, $r = 2$, lower bound

The way we prove this, we create an auxiliary graph, $\Gamma$. 
Hex grid, $r = 2$, lower bound

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- $V(\Gamma) = C$, 

Note: The other grid results use more knowledge to bound $e(\Gamma)$. 

Hex grid, $r = 2$, lower bound

The way we prove this, we create an auxiliary graph, $\Gamma$.

- $V(\Gamma) = C$,
- $\{c, c'\} \in E(\Gamma)$ iff $\exists v \in V(\Gamma)$ such that $C \cap N_2[v] = \{c, c'\}$.

\[|C| + 2e(\Gamma) + 3(n - |C| - e(\Gamma)) \leq 10|C|\]

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**Note:**

We actually consider $\Gamma$ to be a graph on $n$ vertices inside ever-expanding balls around the origin.
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Lemma

$max \ deg(\Gamma) \leq 6 \implies 2e(\Gamma) \leq 6|C|.$
Hex grid, $r = 2$, lower bound

The way we prove this, we create an auxiliary graph, $\Gamma$.

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**Lemma**

$max \ deg(\Gamma) \leq 6 \implies 2e(\Gamma) \leq 6|\mathcal{C}|$.

Now we count the number of pairs $(v, c)$ such that $c \in \mathcal{C} \cap N_2[v]$.

$$|\mathcal{C}| + 2e(\Gamma) + 3(n - |\mathcal{C}| - e(\Gamma)) \leq 10|\mathcal{C}|$$
Hex grid, $r = 2$, lower bound

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$$3n - e(\Gamma) - 2|C| \leq 10|C|$$
Hex grid, \( r = 2 \), lower bound

The way we prove this, we create an auxiliary graph, \( \Gamma \).

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\end{array}
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Now we count the number of pairs \((v, c)\) such that \( c \in C \cap N_2[v] \).

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\[
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A code by any other name...

Stanton also compared Hamming codes in the hypercube to vertex-identifying codes in square lattices:

**Theorem (Stanton 2010++)**

Let $k$ be an integer with $2^k \leq n + 1$. If $D(G)$ denotes the smallest density of a 1-vertex identifying code in $G$, then

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$n = 3, \ D(\mathbb{Z}^3) = 1/4.$
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In particular, if $n = 2^k - 1$, then $D(\mathbb{Z}^n) = \frac{1}{n+1}$.

The upper bound is constructed by finding Hamming codes in an appropriate-dimensional hypercube.
Open questions

• Can the bounds on the 1-identifying code on the hex grid be improved?

\[ \frac{12}{29} \leq D(\mathbb{H}) \leq \frac{3}{7}. \]
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• There are generalizations of codes that distinguish between all pairs of subsets of vertices such that the subsets of size at most \(\ell\). What is the smallest density of such a \((r, \ell)\)-identifying code in the various grids?
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- Many basic questions about vertex identifying codes are unexplored.
Thanks!

My home page:

http://orion.math.iastate.edu/rymartin

Brendon Stanton’s home page:

http://www.public.iastate.edu/~bstanton/

My CV (with links to this and previous talks):

http://orion.math.iastate.edu/rymartin/cv/cv.pdf

Contact me: rymartin@iastate.edu
Contact Brendon: bstanton@iastate.edu