Ex.8 (book). A jet airliner, flying due east at 500 mph in still air, encounters a 70-mph tailwind blowing in the direction 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?

Let \( \vec{u} \) = velocity of the plane
\[ |\vec{u}| = 500 \quad \vec{u} = \langle 500, 0 \rangle \]
\[ |\vec{v}| = 70 \quad \vec{v} = \langle 35, 35\sqrt{3} \rangle \]

\[ \vec{u} + \vec{v} = \langle 535, 35\sqrt{3} \rangle \]

Ground Speed \[ |\vec{u} + \vec{v}| = \sqrt{532^2 + 35^2} \approx 538.4 \text{ mph} \]

New Direction \( \alpha \):
\[ \tan \alpha = \frac{35\sqrt{3}}{535} \]
\[ \alpha = \tan^{-1}\left(\frac{35\sqrt{3}}{535}\right) \approx 6.5^\circ \]
Example (like Ex.9 p. 689). A 100-lb weight hangs from two wires. Find the forces (tensions) \( \vec{F}_1, \vec{F}_2 \) acting on the wires.

Let \( \overrightarrow{w} \) denote the weight of the object.

Magnitude of \( \vec{F}_1 \) is \( |\vec{F}_1| \), \( \vec{F}_1 = \begin{pmatrix} |\vec{F}_1| \cos 50^\circ \\ |\vec{F}_1| \sin 50^\circ \end{pmatrix} \}

Magnitude of \( \vec{F}_2 \) is \( |\vec{F}_2| \), \( \vec{F}_2 = \begin{pmatrix} |\vec{F}_2| \cos 32^\circ \\ |\vec{F}_2| \sin 32^\circ \end{pmatrix} \}

At equilibrium we have: \( \vec{F}_1 + \vec{F}_2 + \overrightarrow{w} = \overrightarrow{0} \)

\( -|\vec{F}_1| \cos 50^\circ + |\vec{F}_2| \cos 32^\circ = 0 \)
\( |\vec{F}_1| \sin 50^\circ + |\vec{F}_2| \sin 32^\circ = 100 \)

We obtain a 2x2 system, solve it to get \( |\vec{F}_1|, |\vec{F}_2| \) & plug into to get \( \vec{F}_1 \) & \( \vec{F}_2 \)

End of 12.2
12.3 The Dot Product.

Sometimes we need to know the angle between two vectors.

For instance, if we needed to find the magnitude of a force $\vec{F}$ in the direction of $\vec{v}$ (we are asking for the component of $\vec{F}$ in the direction of $\vec{v}$)

If we knew the angle $\theta$, then $\vec{F}_v = |\vec{F}| \cos \theta$

Definition: Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ be vectors, then the dot product of $\vec{u}$ and $\vec{v}$ is the scalar defined by:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

We define the angle $\theta$ between two vectors $\vec{u}$ and $\vec{v}$ to be the angle such that

$$0 \leq \theta \leq \pi$$
Properties of the Dot Product (pg. 695) If \( \vec{u} \), \( \vec{v} \) and \( \vec{w} \) are vectors and \( c \) is a scalar, then:

1. \[ \vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u} \]
2. \[ (c \vec{u}) \cdot \vec{v} = \vec{u} \cdot (c \vec{v}) = c(\vec{u} \cdot \vec{v}) \]
3. \[ \vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w} \]
4. \[ \vec{u} \cdot \vec{u} = |\vec{u}|^2 \]
5. \[ \vec{0} \cdot \vec{u} = 0 \]

To find the angle between two vectors...

We use the Law of Cosines:

\[ |\vec{u} - \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2 |\vec{u}| |\vec{v}| \cos \theta \]

We also know:

\[ |\vec{u} - \vec{v}|^2 = (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) \]
Note that if \( \vec{u} \) and \( \vec{v} \) are perpendicular (orthogonal), then

Conversely if \( \vec{u} = 0 \) and \( \vec{v} = 0 \) (so that \( |\vec{u}|, |\vec{v}| \) are nonzero), and \( \vec{u} \cdot \vec{v} = 0 \), then we must have that

Thus \( \vec{v} \) is perpendicular to \( \vec{u} \).

In other words: \( \vec{u} \) and \( \vec{v} \) are orthogonal if and only if \( \vec{u} \cdot \vec{v} = 0 \).

**Projections**

The vector \( \overrightarrow{PR} \) is called the vector projection of \( \vec{u} \) onto \( \vec{v} \).

We denote it by:

Its direction is:

Its length is:

So
Work  The work done by a constant force $F$ in moving an object a distance $d$ is given by:

$$W = Fd$$

But this is only valid when the force is directed along the line of motion of the object.

Example  A tow truck drags a stalled car along a road, the chain makes an angle of 30° with the road and the tension in the chain is 1500N. How much work is done by the truck in pulling the car 1km?