12.1 Three-Dimensional Coordinate Systems.

**2-D space**

2. Q11
   - y-axis
   - ordered pairs: \((x, y)\)

3. Q1II
   - x-axis

4. Q1III
   - ordered pairs: \((x, y)\)

5. Q1IV

**3-D space**

"right-handed" coordinate frame

- \(P\) \((x, y, z)\) ordered triple

- \(x\)-axis
- \(y\)-axis
- \(z\)-axis

\(P(3, 2, 5)\)

**In 3-space we have:**

**coordinate axes:**
- \(x\)-axis
- \(y\)-axis
- \(z\)-axis

**coordinate planes:**
- \(xy\)-plane \((z = 0)\)
- \(xz\)-plane \((y = 0)\)
- \(yz\)-plane \((x = 0)\)

The coordinate planes divide space into 8 cells called **octants**

The 1st octant is the set of points \((x, y, z)\) where \(x > 0, y > 0, z > 0\)

**Interpret Geometrically the following equations & inequalities**

- \(z = 2\)
- \(x = y\)
- \(z \geq 0\) (half-space)
- \(x = 3\)
- \(z = 0, x \leq 0, y \geq 0\)

- Plane through \((0, 0, 2)\), parallel to \(xy\)-plane
- Plane perpendicular to \(xy\)-plane & through \(y = x\) on \(xy\)-plane
- All points above the \(xy\)-plane
- Plane \(y = -3\) to \(yz\)-plane
- Plane \(x = -3\)
Second quadrant on the $xy$-plane.

- $z = 0$, $x \leq 0$, $y \geq 0$

Slab between planes $y = -1$ and $y = 1$ parallel to the $xy$-plane.

- $-1 \leq y \leq 1$

- $y = -2$, $z = 2$

$\{ (x, -2, 2) : x \in \mathbb{R} \}$, a line parallel to the $x$-axis and through the point $(0, -2, 2)$.

What points satisfy $x^2 + y^2 = 4$ and $z = 3$?

Circle center: $(0, 0, 3)$, radius = 2 lies on the plane $z = 3$.

Distance Between 2 points

In 2-D the distance between $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$ is given by: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$C^2 = \Delta x^2 + \Delta y^2$

$C = \sqrt{\Delta x^2 + \Delta y^2}$
In 3-D:

Let \( P_1(x_1, y_1, z_1) \) & \( P_2(x_2, y_2, z_2) \)

\[
C^2 = \Delta x^2 + \Delta y^2
\]

\[
D^2 = C^2 + \Delta z^2
\]

\[
D^2 = \Delta x^2 + \Delta y^2 + \Delta z^2
\]

\[
|P_1P_2| = D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}
\]

E.g. Find the equation of a sphere with radius \( r \) and center \((h, k, l)\).

A sphere is the set of points @ a distance \( r \) away from the center so a point on this sphere \((x, y, z)\) satisfies:

\[
r = \sqrt{(x-h)^2 + (y-k)^2 + (z-l)^2}
\]

Also → \( r^2 = (x-h)^2 + (y-k)^2 + (z-l)^2 \)

* Show that \( x^2 + y^2 + z^2 = 4x - 2y \) represents a sphere.

Complete Squares:

\[
(x^2 - 4x + 4) + (y^2 + 2y + 1) + z^2 = 5
\]

\[
(x-2)^2 + (y+1)^2 + z^2 = 5 = r^2
\]

Sphere centered @ \((2, -1, 0)\) and radius is \( \sqrt{5} \)