For each problem that uses Matlab or some other tool, you should hand in a printout of the relevant script or function file(s), or a transcript of your interactive session, plus whatever outputs or plots are requested. Put the problems in the proper order, and label all printouts clearly. The final output should have full accuracy (format long); intermediate results can be shorter, if you want.

1. Write a Matlab script to product graphs of the functions \( y = 3 \sin(\pi x) \) and \( y = e^{-0.2x} \) on the same graph, for \( x = 0 : 0.02 : 4 \). Label both axes and the graph as a whole. Use \texttt{gtext} to label one of the intersection points of the graphs. (10)

2. Write a Matlab function \texttt{wilk} so that \texttt{wilk}(n) produces the Wilkinson matrix of size \( n \). This is an \( n \times n \) matrix that has the numbers \( n, n-1, \ldots, 1 \) on the diagonal, \( n \) above the diagonal, and zeros everywhere else. For example,

\[
\text{wilk}(3) = \begin{pmatrix}
3 & 3 & 0 \\
0 & 2 & 3 \\
0 & 0 & 1
\end{pmatrix}.
\]

Don’t use any loops. Hint: the \texttt{diag} function.

After you have it working as a regular function, rewrite it as an inline function. Demonstrate your programs for \( n = 1, 2, 3, 4 \).

Note: Wilkinson was apparently a busy guy. After someone in a previous class pointed this out, I discovered that there are at least another 5 definitions of “Wilkinson matrix”. (10)

3. (a) Polynomials are represented in Matlab by their coefficient vectors (highest power first). For example, the polynomial \( x^2 - x + 2 \) is represented by the vector \([1, -1, 2] \).

For the polynomials

\[
p(x) = 5x^4 + 3x^3 - x^2 + 1 \\
q(x) = x^2 - x + 2
\]

use Matlab’s polynomial functions (\texttt{help polyfun}) to compute the following:

- \( p(x) \cdot q(x) \)
- \( p(x)/q(x) \) (as polynomial part + remainder)
- \( p'(x) \)

This will only take one line each.

(b) Write a function \texttt{polyadd} which adds two polynomials, and use it to add \( p(x) + q(x) \).

(c) (2 points extra credit) Write \texttt{polyadd} as an inline function. (10)

4. IEEE single precision floating point numbers have 24 bits of mantissa. What is the largest factorial that can be represented exactly?

Caution: this is harder than it looks. You have to think in terms of what goes into the mantissa, and what goes into the exponent. (10)
5. It is well-known that the solutions of the quadratic equation \( ax^2 + bx + c = 0 \) are given by

\[
x_{1,2} = \frac{1}{2a} \left( -b \pm \sqrt{b^2 - 4ac} \right).
\]

Under what conditions do you expect bad accuracy due to cancellation in this formula? Hint: cancellation does not always lead to bad accuracy.

For the case where harmful cancellation does occur, use algebraic manipulation to come up with an alternative formula that is more accurate.

Try both formulas on the equation \( x^2 - 10^5 x + 1 = 0 \). Assuming that the more accurate formula gives the exact solution, what relative error does the less accurate formula have?

Matlab has a function `roots` built in, for solving polynomial equations. For the given equation, compare its results to yours, and comment.