(1) [3pts.] For positive real numbers $x, y$, show that
\[ \sqrt{xy} \leq \frac{x + y}{2}. \]

(2) [5pts.] For real numbers $x, y$, suppose $y - x > 1$. Prove:
\[ \exists n \in \mathbb{Z}. \quad n \in (x, y). \]

(3) [3pts.] Find a number $M$ such that $|x^3 - 5x^2 + 2x| \leq M$ for all $-3 \leq x \leq 1$.

(4) [3pts.] For a sequence $\{x_n\}$, suppose that the subsequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are convergent. Show that $\{x_n\}$ is convergent if and only if $\lim_{n \to \infty} x_{2n} = \lim_{n \to \infty} x_{2n+1}$. 