(1) Consider the functions $f, g: \mathbb{R} \to \mathbb{R}$ defined by
\[ f(x) = \sqrt{x^2 + 1} \quad \text{and} \quad g(x) = x^2. \]
Find the formulas for $f \circ g$ and $g \circ f$.

(2) Find the inverse of the function $f: \mathbb{R} \setminus \{-\frac{1}{2}\} \to \mathbb{R} \setminus \{\frac{5}{2}\}$ with
\[ f(x) = \frac{5x + 2}{2x + 1}. \]

(3) Prove or disprove the following:
**Claim.** Suppose that $A_i$ is an uncountable set for each natural number $i$. Then $\bigcup_{i \in \mathbb{N}} A_i$ is uncountable.

(4) Prove or disprove the following:
**Claim.** Suppose that $X$ is a countably infinite set, and $Y$ is an uncountable set. Then if $X \subseteq Z \subseteq Y$ (so $X$ properly contained in $Z$), the set $Z$ is uncountable.