(1) [3pts.] For positive real numbers $x, y$, show that
\[\sqrt{xy} \leq \frac{x + y}{2}.\]

(2) [4pts.] For real numbers $x, y$, suppose $y - x > 1$. Prove:
\[\exists n \in \mathbb{Z}. \ n \in (x, y).\]

(3) [3pts.] Find a number $M$ such that $|x^3 + x^2 - 2x| \leq M$ for all $-1 \leq x \leq 2$.

(4) [3pts.] Is the sequence $\left\{ \frac{1}{\sqrt{n}} \right\}$ convergent? If so, what is the limit? (Justify your answers.)