1. Find the general solution of

\[(1 + x^2)u_x + u_y = 0.\]

Sketch some of the characteristic curves.

2. Find the solution of

\[yu_x + xu_y = 1, \quad u(0, y) = e^{-y^2}.\]

Discuss why the solution you find is only valid for \(|y| \geq |x|\).

3. Find the general solution of \(u_{xx} - 4u_{xy} + 3u_{yy} = 0\).

4. Find the regions of the \(x - y\) plane where the PDE

\[yu_{xx} - 2u_{xy} + xu_{yy} - 3u_x + u = 0\]

is elliptic, parabolic, and hyperbolic.

5. Carry out the details of showing that in polar coordinates \((r, \theta)\) Laplace’s equation in \(\mathbb{R}^2\) becomes

\[u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0.\]