Show your work and explain your answers; Circle your final answer

1. (a) (10 points) Find the general solution of the differential equation
   \[ y' + \frac{y}{x+2} = x \]

(b) (10 points) Find the solution of the initial value problem
   \[ y' = xe^y\sqrt{x^2 + 9} \quad y(4) = 0 \]
(c) (15 points) Find the general solution of

\[ 2y'' + y' - 3y = x + e^{2x} \]
2. (a) (15 points) Consider the homogeneous system of equations

\[ \dot{x} = A \dot{x}, \]

where \( A \) is the matrix

\[
A := \begin{pmatrix}
1 & 1 & 0 \\
0 & 1 & 0 \\
0 & 1 & 1 \\
\end{pmatrix}
\]

Find the general solution.
(b) (10 points) Find the general solution of the system

\[ \dot{x} = A \tilde{x}, \]

where

\[ A := \begin{pmatrix} 2 & -2 \\ 4 & -2 \end{pmatrix}. \]
3. (10 points) Consider the Initial Value Problem

\[ \ddot{x} = A \dot{x} + \dot{f}, \]

\[ A := \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \dot{f} = \begin{pmatrix} t e^t \\ e^t \end{pmatrix}, \quad x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \]

Calculate the solution of this Initial Value Problem.

4. (a) (5 points) Find the inverse Laplace transform of

\[ F(s) = \frac{3}{s^2 - 5s + 4} \]
(b) (3 points) Find the Laplace transform of the function
\[ f(t) - \int_0^t \sin(2\tau) \cos(t - \tau) d\tau, \]

(c) (4 points) Find the Laplace transform of the function \( f(t) \) where \( f(t) \) is the function equal to \( f(t) = 1 \), when \( 0 \leq t < 1 \) and equal to \( f(t) = t \) when \( 1 \leq t < \infty \).
(d) (3 points) Find the Laplace transform of a periodic function \( f = f(t) \) which is periodic of period 1 and it is equal to 3, for \( t \) in \([0, \frac{1}{2}]\) and zero for \( t \) in \((\frac{1}{2}, 1)\).

5. (15 points) Solve the following initial value problem by using the Laplace transform formalism

\[ y'' + 4y = 2 + \delta(t - \pi), \quad y(0) = 1, \quad y'(0) = 0 \]