1. Find the volume of the solid generated by revolving about the $x$-axis the region bounded by $x = \sqrt{y} + 1$, $y = 4$, $y = 0$ and $x = 0$.

**Solution:** Use the shell method.

\[
V = 2\pi \int_0^4 y(\sqrt{y} + 1) \, dy \\
= 2\pi \int_0^4 \left( y^{3/2} + y \right) \, dy \\
= 2\pi \left[ \frac{2}{5} y^{5/2} + \frac{1}{2} y^2 \right]_0^4 \\
= 2\pi \left[ \frac{64}{5} + 8 \right] = \frac{208\pi}{5}
\]
2. Set up **but do not evaluate** an integral that computes the length of the curve

\[ R(t) = (t^{3/2}, t^{5/2}) \]  
from \( t = 0 \) to \( t = 5 \).

**Solution:**

\[
\int_0^5 \sqrt{\left(\frac{3}{2} t^{1/2}\right)^2 + \left(\frac{5}{2} t^{3/2}\right)^2} \, dt = \int_0^5 \frac{1}{2} \sqrt{9t + 25t^3} \, dt \approx 57.4441.
\]
3. Assume the figure below is the cross-section of a 25 ft. long right triangular wedge filled with water:

![Cross-section of the tank in Problem 3.](image)

Figure 1: Cross-section of the tank in Problem 3.

Setup **but do not evaluate** the integral to calculate the work done by pumping the water 4 ft. above the top of the tank. **Note:** The density of water is \( \delta = 62.4 \text{ lb. per cubic foot.} \)

**Solution:**

The equation of hypotenuse is \( y = \frac{6}{4}x = \frac{3}{2}x \), so \( x = \frac{2}{3}y \).

The width of the triangle cross-section is \( w = x = \frac{2}{3}y \).

Each slab contributes \( \Delta W = \delta \Delta V \cdot h = \delta \cdot 25 \left( \frac{2}{3}y \right) (10 - y) \Delta y \).

The total work is \( W = \delta \int_0^6 25 \left( \frac{2}{3}y \right) (10 - y) dy = \frac{50 \delta}{3} \int_0^6 (20y - y^2) dy \).

\(^1\)This was an error in the exam. It said “square”. 
4. Find the centroid of the region bounded by the curve $y = 4 - x^2$ and the $x$-axis.

Solution:

$$m = \int_{-2}^{2} (4 - x^2) \, dx = \left[ 4x - \frac{1}{3}x^3 \right]_{-2}^{2} = \frac{32}{3}$$

$$M_y = \int_{-2}^{2} 4x - x^3 \, dx = \left[ 2x^2 - \frac{1}{4}x^4 \right]_{-2}^{2} = 0$$

$$\bar{x} = \frac{M_y}{m} = 0 \text{ (by symmetry)}$$

$$M_x = \int_{-2}^{2} \frac{1}{2} (4 - x^2)^2 \, dx = \frac{1}{2} \int_{-2}^{2} (16 - 8x^2 + x^4) \, dx = \frac{1}{2} \left[ 16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_{-2}^{2} = \frac{256}{15}$$

$$\bar{y} = \frac{M_x}{m} = \frac{8}{5}$$