Problem 9. For real number $a$, let $\lfloor a \rfloor$ denote the largest integer less than or equal to $a$, and let $\{a\}$, the fractional part of $a$, be defined by $\{a\} = a - \lfloor a \rfloor$. As examples, $\lfloor 3.6 \rfloor = 3$, $\{3.6\} = 0.6$, $\lfloor -3.6 \rfloor = -4$, and $\{ -3.6 \} = 0.4$. Find all real number solutions $(x, y, z)$ to the system

\[
\begin{align*}
x + \lfloor y \rfloor + \{z\} &= -7.1 \\
\lfloor x \rfloor + \{y\} + z &= 18.8 \\
\{x\} + y + \lfloor z \rfloor &= 4.7.
\end{align*}
\]

Solution. We use the fact that for any real number $a$, $a = \lfloor a \rfloor + \{a\}$, which is equivalent to $a - \{a\} = \lfloor a \rfloor$.

In the original system of equations, add the first two equation and subtract the third, keeping in mind the relations just mentioned:

\[
(x + \lfloor x \rfloor - \{x\}) + (\lfloor y \rfloor + \{y\} - y) + (z + z - \lfloor z \rfloor) = (-7.1 + 18.8 - 4.7),
\]

leading to

\[
\lfloor x \rfloor + \{z\} = \frac{7}{2} = 3.5.
\]

Because $\lfloor x \rfloor$ is an integer and $\{z\}$ is a positive number in $[0, 1)$, it follows that $\lfloor x \rfloor = 3$ and $\{z\} = 0.5$.

By similar reasoning, adding the first and third of the above three equation, then subtracting the second equation leads to

\[
\{x\} + \lfloor y \rfloor = -10.6 = -11 + 0.4 \quad \text{from which} \quad \{x\} = 0.4 \quad \text{and} \quad \lfloor y \rfloor = -11.
\]

Adding the second and third equations and subtracting the first we find

\[
\{y\} + \lfloor z \rfloor = 15.3 \quad \text{from which} \quad \{y\} = 0.3 \quad \text{and} \quad \lfloor z \rfloor = 15.
\]

We can now recover $x, y, z$ by using $a = \lfloor a \rfloor + \{a\}$ to see

\[
x = 3.4, \quad y = -11 + 0.3 = -10.7 \quad z = 15.5
\]