**Problem 14.** One hundred passengers board a plane that seats 100. Each passenger has an assigned seat. The first two passengers that board ignore the seat assignments and each chooses a seat at random. After that, each subsequent passenger does take their assigned seat unless that seat is already occupied, in which case the passenger takes a random empty seat. What is the probability that the last passenger aboard gets his or her assigned seat?

**Solution.** The probability that the 100\textsuperscript{th} passenger gets his or her assigned seat is \(\frac{1}{3}\).

For convenience, assume that passenger \(k\) is assigned to seat \(k\). When passenger 100 (the last passenger) gets on the plane there is one empty seat and that seat must be seat 1, 2, or 100. In particular, when passenger 100 enters the plane, none of the seats 3—99 can be empty; if seat \(k\) were empty for some 3 \(\leq k \leq 99\), this would mean that passenger \(k\) did not sit in his/her assigned seat, contradicting the hypotheses of the problem. Because anyone sitting in seat 1, 2, or 100 chose that seat randomly from the among the unoccupied seats, each of 1, 2, or 100 is equally likely to be the last seat.