**Problem 12.** Triangle $ABC$ with $A = (11, 27)$, $B = (180, 1)$, and $C = (102, 66)$ is rotated in the plane about a point $P$. The result is triangle $A'B'C'$ with $A' = (17, -81)$, $B' = (-24, 85)$, and $C' = (-54, -12)$. Find the coordinates of point $P$.

**Solution.** The point $P$ has coordinates $(50, -25)$.

Because $A$ and $A'$ are equidistant from $P$, the point $P$ must lie on the perpendicular bisector of $AA'$. The equation for this perpendicular bisector is

$$x - 81y = 500.$$ 

The point $P$ also lies on the perpendicular bisector of $BB'$ and on the perpendicular bisector of $CC'$. The equations for these are

$$17x - 7y = 1025 \quad \text{and} \quad 2x + y = 75,$$

respectively. Solving these three equations simultaneously we find a unique solution (the three lines are concurrent) $x = 50$ and $y = -25$. It follows that $P = (50, -25)$.