**Problem 11.** The figure below shows two congruent circles centered at $A$ and $B$. If $\angle PFE = 75^\circ$, then what is the measure of angle $EPF$?

![Diagram of two congruent circles centered at A and B with a triangle PFE]

**Solution.** The measure of angle $EPF$ is $10^\circ$.

Draw segments $AB$ and $BE$ and note that the former is a radius in circle $A$ and that both are radii in circle $B$. Let the measure of angle $EPF = x$. Then because $AP = AB$, we find that $\angle ABP = x$. Since $\angle BAE$ is an exterior angle to triangle $ABP$, we have

$$\angle BEA = \angle APB + \angle ABP = 2x.$$

and it then follows that also $\angle AEB = 2x$. Next note that because $EB = FB$, we have $\angle BEF = \angle AFE = 75^\circ$, and hence $\angle EBF = 30^\circ$. Because $\angle EBF$ is an exterior angle to triangle $BEP$, we have

$$30^\circ = \angle EBF = \angle BEP + \angle BPE = 2x + x = 3x,$$

so $10^\circ$. 

![Diagram showing the angles and segments labeled with $x$ and $2x$]