Problem 1. In the town of Resortville the high state and local sales taxes combine to give a total sales tax of 12% on any purchase. Marlisse buys a fast food lunch at a Resortville eatery and notices that her bill, including tax, is an exact number of dollars (with no cents) and that the sales tax did not have to be rounded. What was the cost, with tax, of Marlisse’s lunch? You must justify your answer with complete work and reasoning.

Solution. Let $C$, in cents, be the before tax cost of Marlisse’s lunch. Then the cost, with tax, is $1.12 \cdot C$. Because this product is an exact number of dollars, it must be divisible by 100. Thus $112 \cdot C = 2^4 \cdot 7 \cdot C$ must be a multiple of 10000. The smallest value of $C$ for which this happens is $C = 5^4 = 625 = \$6.25$ and the second largest value is $2 \cdot 5^4 = 1350 = \$13.50$. Because this is a “fast food lunch” we assume the cost is under $10.00, so the pre-tax cost of the lunch is $\$6.25$. The with tax cost is $1.12(\$6.25) = \$7.00$.

Note. Actually any before tax cost that is a positive integer multiple of $\$6.25$ results in a with tax cost that is, without rounding, a whole number of dollars. The description of this meal as a “fast food meal” was to direct students to the least pricey answer of $\$7.00$ after tax. However, we did accept as correct any positive integer multiple of $\$7.00$ as correct as long as there was supporting work (even guess and check) to justify the answer. The record for the most expensive fast food meal is $\$112.00$. 