Problem 9. On a $10 \times 10$ checkerboard, a real number is written in each square (exactly one number to a square.) It turns out that in each square, the number in the square is the average of the numbers in its the neighboring squares. (The neighbors of a square $S$ are all squares that touch $S$, so a square in the interior of the board has 8 neighbors, a corner square has 3 neighbors, etc.) Prove that the numbers in the squares are all the same.

Solutions due by 10:00am Monday, March 23, 2015.