Problem 4. A UPS fleet consist of 400 diesel trucks. One very cold morning, only one of the 400 trucks would start. Thinking quickly, they moved this one truck so its exhaust would warm another truck. Suppose that it takes

a. 10 minutes for a truck to warm up before it can be moved.

b. 10 minutes to position a running truck to warm another.

c. 10 minutes for the exhaust from a running truck to warm a cold truck.

From the instant that the first truck is started, how long will it take to start all of the trucks?

Solution 4. We count trucks that are in one of three mutually disjoint states: starting, moving, and warming another truck. Let $s[t]$ be the number of trucks that are started at time $10t$ minutes, (and must idle for ten minutes before being moved.) Let $m[t]$ be the number of trucks that start moving at time $10t$ and are in motion for ten minutes in positioning them to warm another truck. Let $w[t]$ be the number of trucks that start warming another truck at time $10t$. Tracking the process through the first 20 minutes we see

$$s[0] = 1, \quad s[1] = 0, \quad s[2] = 0$$

$$m[0] = 0, \quad m[1] = 1, \quad m[2] = 0$$

$$w[0] = 0, \quad w[1] = 0, \quad w[2] = 1.$$ 

For $t \geq 3$ we have the recursions relationships,

$$s[t] = h[t - 1], \quad m[t] = s[t - 1] + h[t - 1], \quad h[t] = m[t - 1].$$

With these initial conditions and recursion formula we can easily track the process of the truck starting party. We obtain the following results (done on Mathematica), showing that all trucks are started at the beginning of the 230-th minute, that is, after 3 hours and 50 minutes.