Problem 15. Let \( f(t) = at^2 + bt + c \) where \( a, b, c \) are nonnegative real numbers. Prove that

\[
(f(xy))^2 \leq f(x^2)f(y^2)
\]

for all real numbers \( x, y \).

Solution 15. This is an immediate consequence of the Cauchy Schwarz Inequality:

For real numbers \( r_j, s_j, 1 \leq j \leq n \),

\[
\left( \sum_{j=1}^{n} r_j s_j \right)^2 \leq \left( \sum_{j=1}^{n} r_j^2 \right) \left( \sum_{j=1}^{n} s_j^2 \right).
\]

Applying this result in the case \( n = 3 \),

\[
(f(xy))^2 = \left( (\sqrt{a}x^2)(\sqrt{a}y^2) + (\sqrt{b}x)(\sqrt{b}y) + \sqrt{c} \right)^2
\leq \left( (\sqrt{a}x^2)^2 + (\sqrt{b}x)^2 + \sqrt{c} \right) \left( (\sqrt{a}y^2)^2 + (\sqrt{b}y)^2 + \sqrt{c} \right)
= f(x^2)f(y^2).
\]