**Problem 7.** There are 21 teams entered in a round-robin tournament. Each team plays other team exactly once, and each team wins exactly 10 games. (There are no ties.) A symmetric triangle is a set of three teams, no one of which beats both of the other two. How many symmetric triangles can there be in this tournament?

**Solution.** Define a triangle to be a set of three different players. Then there are \( \binom{21}{3} \) triangles. If a triangle is not symmetric, then there is one player in the triangle who beats the other two. Given a player \( A \), because \( A \) beats 10 other people, \( A \) will be the dominant player in \( \binom{10}{2} \) triangles. Because a non-symmetric triangle has exactly one dominant player, there are a total of \( 21\binom{10}{2} \) such triangles. All other triangles are symmetric. Thus there are

\[
\binom{21}{3} - 21\binom{10}{2} = 385
\]

non-symmetric triangles.