Problem 12. Your friend and you play a game with the following rules: If one of you says an integer $n$, the other then says an integer of their choice between $2n$ and $3n$, inclusive. Players alternate turns. Whoever first says 2009 or greater loses the game, and their opponent wins. You must begin the game by saying a positive integer less than 10. With how many of them can you guarantee a win?

Solution. There are 6 such numbers. The easiest way to analyze the game is to work backwards. Certainly I win if I say any number between 1004 and 2006. This also means my friend can force a win for himself if I ever say a number between 335 and 1003. Then I will win if I say any number between 168 and 334, because my friend must then say one of the losing numbers between 335 and 1003. Continuing with this reasoning, I lose if I say 56 through 167, win if I say 28 through 55, lose with 10 through 17, win with 5 through 9, lose with 2 through 4, and win with 1.