Problem 6. Wayne walks at a constant rate of 2 meters per second. His wife Jody jogs at a constant rate of 4 meters per second, and their dog Runnels runs at 6 meters per second. During their exercise period, when Wayne walks and Jody jogs, Runnels entertains himself by repeatedly running from Wayne to Jody and back again. One Sunday morning Wayne and Jody walk and jog, respectively down a long street. Jody starts 2 meters in front of Wayne. If Runnels starts from Wayne’s starting point at the start of the exercise period, how far will Runnels have run when he returns to Wayne for the tenth time?

Solution. At time $t = 0$, the start of the run, let Wayne and Runnels be at position $d = 0$ and Jody’s position be $d = 2$. The figure above shows the distance time graphs for Wayne, Jody, and Runnels. The graph shows that Runnels first catches Jody at time $t = 1$ and returns for the first time to Wayne at time $t = 3/2$, indicated by point $B'$. The point $C'$ indicates that at this time Jody is at position $d = 8$.

Runnels returns to Wayne for the second time at the time indicated by $A''$. Note that $BCC'B'$ is similar to $B'C'C'B''$ and that $BC = 2$ and $B'C' = 5$. It follow by similarity that $A'A'' = \frac{5}{2}AA'$ and that $B''C'' = \frac{5}{2}B'C'$. It follows that each trip from Wayne to the next return to Wayne takes Runnels $\frac{5}{2}$ as long as the previous such trip. Thus, by the time Runnels returns to Wayne for the tenth time he will have run for

$$\frac{3}{2} + \frac{3}{2}\left(\frac{5}{2}\right) + \frac{3}{2}\left(\frac{5}{2}\right)^2 + \frac{3}{2}\left(\frac{5}{2}\right)^3 + \cdots + \frac{3}{2}\left(\frac{5}{2}\right)^9 = \frac{3}{2} \sum_{k=0}^9 \left(\frac{5}{2}\right)^k$$

It in this time Runnels’ runs a total distance of

$$6\left(\frac{3}{2} \sum_{k=0}^9 \left(\frac{5}{2}\right)^k\right) = 9\left(\frac{5}{3}\right)^{10} - 1 \left(\frac{5}{2} - 1\right) = 57214.458984375$$