Problem 2. There is a list of 101 statements on a page, numbered 1, 2, . . . , 101. Statement number \( k \) says

There are more than \( k \) false statements in this list.

Which of the 101 statements are true? You must include your reasoning with your answer.

Solution. Statements 1–50 are true and statements 51-101 are false.

We consider the statements one at a time, starting with the last. The statement “There are more than 101 false statements in this list” is clearly false because the list only has 101 statements. Likewise the statement “There are more than 100 false statements in this list” is false: if it were true then there would be at most \( 101 - 1 = 100 \) false statements in the list and the statement would be self-contradictory. Thus the last two statements it must be false, and hence Statement 1 is true. Now consider Statement 99. If it were true then there would be at most \( 101 - 2 = 99 \) false statements making this statement self-contradictory. Therefore Statement 99 is false and it now follows that Statement 2 is true. Repeating this reasoning process we find that for \( 1 \leq j \leq 50 \), Statement \( 101 - j \) is false and statement \( j \) is true.