Problem 1. You are given a 12 inch stick of wood and asked to make a “ruler” that can measure any whole number inch distance from 1 inch to 12 inches. What is the minimal number of marks that you need to make on the ruler to do this? For example, if you have a mark at the 3 inch position and another at the 8 inch position, then you can measure 5 inches by measuring from the 3 inch mark to the 8 inch mark. Produce a “ruler” with the minimal number of marking that can be used for the required measurements. Give a convincing argument that your ruler does indeed have the fewest possible markings. For example, the ruler with four marks shown below can be used to measure 3, 4, 5, 8, 9, 12 inches.

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 0  3  8  12
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Solution. It is not hard, by trial and error to come up with a 6 mark ruler that does the job (here we are count the two ends of the ruler as two of the marks.) An example of such a ruler is

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 0  1  4  5 10 12
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To see that no fewer than 6 marks can suffice, note that if the ruler had five marks (two of which are the endpoints) then the number of different distances that can be measured is at most \( \binom{5}{2} = 10 \), which is not enough possibilities to measure 12 distinct distances.

Note: Many students found a workable ruler with 6 marks but did not explain why no such ruler with 5 marks was possible.