Problem 8. In a school with 100 students, each student sends a greeting card to 50 other students. Prove that there is a pair of students who send a card to each other.

Solution 8. First note that there are
\[
\binom{100}{2} = \frac{100 \cdot 99}{2} = 4950
\]
different pairs of students. Also, the number of cards sent is
\[
100 \cdot 50 = 5000.
\]
If for each pair, only one card was sent, then at most 4950 cards could be sent. Thus there must be pairs that accounted for two cards, e.g., pairs for which each student sent a card to the other. In fact, there are at least \(5000 - 4950 = 50\) such pairs.