Problem 9. Given a positive integer \( n \), let \( S(n) \) denote the sum of the digits in the base-ten representation of \( n \). For example, \( S(237) = 2 + 3 + 7 = 12 \) and \( S(5) = 5 \). Define the sequence \( a_k, k = 1, 2, 3, \ldots \) by

\[
a_1 = 1 \quad \text{and} \quad a_{k+1} = a_k + S(a_k),
\]

for \( k = 1, 2, 3, \ldots \). Is there an index \( k \) such that \( a_k = 99887766554433221100 \)? If so what is the \( k \)? If not, justify your answer.