Problem 6. An elevator in a 66 story apartment building works strangely. When the UP button is pressed, the elevator goes up exactly 8 floors, and does not move at all if there are fewer than 8 floors above its starting location. When DOWN is pressed, the elevator drops exactly 11 floors, but again does not move if there are fewer than 11 floors below the starting location. Is it possible to take this elevator from any floor to any other floor?

Solution 6. Yes, every floor can be reached. Students took many approaches to prove this. One solution is to note that if we can get to each of the floors 1, 2, 3, 4, 5, 6, 7, 8, the we can reach any floor. In particular if the number of the desired floor is $8k + r$, $1 \leq r \leq 8$, then go to floor $r$ and then press the up button $k$ times. To get to floor $r$, use the down button until you are on floor $f$, with $10 \leq f \leq 20$. Now if the up button is pressed 4 times and then the down button 3 times, then the elevator will be at floor $f - 1$. Repeat this process to drop one floor at a time until at floor $r$.

Note: If the building has 18 or fewer floors, then there are floors that cannot be accessed. For example, if in the 18 story building the elevator is on floor 11, then the elevator cannot move. However, it is not hard to show that with 19 floors every floor can be reached and it then easily follows that for any buildings of more than 18 floors, the elevator can get to any floor.