Math 165 Midterm

Part I

Spring 2015

Part I of the exam has 6 problems. The point value for each problem is indicated next to the problem. You may NOT use a calculator on this section. You must show all work. No credit is allowed for mere answers with no work shown. Show the steps of calculations. State the reasons that justify conclusions. This part of the exam will be collected after 45 minutes.

Question 1 __________

Question 2 __________

Question 3 __________

Question 4 __________

Question 5 __________

Question 6 __________

Total Points __________
Part I of the exam has 6 problems. The point value for each problem is indicated next to the problem. You may NOT use a calculator on this section. You must show all work. This part of the exam will be collected after 45 minutes.

1. (15 points) Find the following limits:
   
   a. \( \lim_{{x \to 5}} \frac{x^2 - 25}{x - 5} \)
   
   b. \( \lim_{{x \to \infty}} \frac{5x + 4}{\sqrt{3x^2 - 5}} \)
   
   c. \( \lim_{{x \to 0}} \frac{\sin 3x}{x} \)
2. (7 points) Find the derivative of \( y = \frac{x(x^2 + 2)}{x - 1} \). (Do not simplify.)

3. (7 points) Find the derivative of \( f(x) = \sin \left[ \cos \left( x^2 \right) \right] \).
4. (8 points) Find $\frac{dy}{dx}$ for $x^3 - 3x^2 y + 2xy^2 = 12$ using Implicit Differentiation.

5. (8 points) Find $\frac{d^3 y}{dx^3}$ for the function $y = 5\sin(2x)$. 
6. (5 points) Below is a graph of $h(x)$. In the blank graph, draw an approximate graph of $h'(x)$. 

![Graph of h(x)](image1.png)

![Graph of h'(x)](image2.png)
Part II of the exam has 6 problems. The point value for each problem is indicated next to the problem. Answer each question completely. You may use calculator. Show all work. No credit is allowed for mere answers with no work shown. Show the steps of calculations. State the reasons that justify conclusions.

Question 1 __________
Question 2 __________
Question 3 __________
Question 4 __________
Question 5 __________
Question 6 __________

Total Points __________
Part II of the exam has 6 problems. The point value for each problem is indicated next to the problem. Answer each question completely. You may use calculator. Show all work. No credit is allowed for mere answers with no work shown. Show the steps of calculations. State the reasons that justify conclusions.

1. (7 points) Find the equation for the line tangent to \( y = \frac{2x + 1}{x^2 + 1} \) for \( x = 0 \).

2. (8 points) Use the definition of derivative to calculate the derivative of the function \( R(x) = \frac{x}{x + 3} \).
3. (7 points) A particle has the position function $s(t) = \frac{1}{2}t^4 - 5t^3 + 12t^2$. Find the velocity of the particle when its acceleration is zero.

4. (8 points) Let $f(x) = \begin{cases} x^2 & \text{if } x \leq -2 \\
ax + b & \text{if } -2 < x < 2 \\
2x - 6 & \text{if } 2 \leq x \end{cases}$.

Find the values of $a$ and $b$ such that $\lim_{x \to -2} f(x)$ and $\lim_{x \to 2} f(x)$ both exist.

5. (8 points) Use logarithmic differentiation to find the derivative of $y = \frac{(x^3 - 1)^4 \sqrt{3x - 1}}{x^2 + 4}$. 
6. (12 points) Let \( f(x) = p(x)q(x) \), \( g(x) = \frac{p(x)}{q(x)} \), and \( h(x) = p(q(x)) \). Use the graphs of \( p(x) \) and \( q(x) \) to do problems (a) through (d) below:

![Graph of \( p(x) \) and \( q(x) \)]

a. Find the value of \( f'(3) \) or explain why it doesn’t exist.

b. Find the value of \( g'(11) \) or explain why it doesn’t exist.

c. Find the value of \( h'(1) \) or explain why it doesn’t exist.