Math 166 Final Exam
Calculators can be used during the exam, but you must justify your answers to get full credit. “I did it on my calculator” is not sufficient justification for any results.

May 2, 2012

Name:________________________

Student ID:____________________

TA/Section:____________________

1. (4 points each) Find each of the following limits or state why it does not exist.

   a) \( \lim_{x \to 1} \frac{\ln x}{x^2 - 1} \).

   b) \( \lim_{x \to \pi/2} (\tan x - \sec x) \).

   c) \( \lim_{x \to 0} (1 + \sin x)^{3/x} \).
2. (4 points each) Evaluate each improper integral, or show that it diverges.

a) \( \int_{-3}^{2} \frac{dx}{x^3} \)

b) \( \int_{3}^{\infty} \frac{dx}{x(\ln x)^2} \)

c) \( \int_{1}^{5} \frac{dx}{\sqrt{x - 1}} \)
3. (4 points each) Determine if each of the following series converges or diverges:

a) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \]

b) \[ \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \]

c) \[ \sum_{k=1}^{\infty} \left( \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}} \right) \]

d) \[ \sum_{k=1}^{\infty} \frac{2n - 3}{1000n^2 + n + 13} \]

4. (12 points) Find the interval of convergence of the power series

\[ \sum_{n=2}^{\infty} \frac{2^n}{\ln n} (x - 1)^n \]
5. (12 points) Find the McLaurin polynomial \( P_6 \) (of degree 6) for the function \( f(x) = \frac{\cos(x^2)}{e^{x^2}} \).

6. (12 points) If \( f(x) = \sqrt{1 + x} \) is approximated by its Taylor polynomial
\[
p_3(x) = 1 + \frac{1}{3} x - \frac{1}{9} x^2 + \frac{5}{81} x^3
\]
estimate the absolute value of the error \( R_3(x) = f(x) - p_3(x) \) for
\[0 \leq x \leq \frac{1}{2} \]

[Hint: Use the formula \( R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x - a)^{n+1} \) for some \( c \) between \( a \) and \( x \).]
7. (12 points) Find the equation of the tangent line to the parametric curve \( x = t^3 - 4, \ y = t^2 + 2, \ 0 \leq t \leq 4, \) at the point (4,6).

8. (12 points) Find the area of the region inside the cardioid \( r = 2 + 2 \sin \theta \) and outside the circle \( r = 3. \)