Math 166: Final Exam
Fall 2015 - Thursday December 17, 2015

The exam has 10 problems for a total of 100 possible points. Each problem is worth 10 points.

Calculators are permitted for this exam. The exam concludes after 2 hours.

Question 1: ______

Question 2: ______

Question 3: ______

Question 4: ______

Question 5: ______

Question 6: ______

Question 7: ______

Question 8: ______

Question 9: ______

Question 10: ______

100 Total Points: ______

You must show all work, but you need not simplify your answers unless instructed to do so.
Question 1. Does the following series converge or diverge? If it converges, find its sum.

\[
\sum_{n=1}^{\infty} \frac{2}{(2n - 1)(2n + 1)}
\]
**Question 2.** Determine if the following series converges or diverges. Indicate your method, and give reasons for your answer.

\[ \sum_{n=1}^{\infty} \frac{n + 20}{\sqrt{n^5 + n^2 + 15/n}} \]

**Question 3.** Determine if the following series converges absolutely, converges conditionally, or diverges. Indicate your method, and give reasons for your answer.

\[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} \]
Question 4. One of the following series converges and one diverges.

Determine which is which. Indicate your method, and give reasons for your answer.

\[
S = \sum_{n=1}^{\infty} \frac{n^{n+1}}{(2n + 1)^n}
\]

\[
T = \sum_{k=0}^{\infty} \frac{(2k)!}{3^k(k!)^2}
\]
Question 5. Consider the following power series

$$\sum_{n=1}^{\infty} \frac{n+1}{n^3 e^n} (x - 2)^n. $$

(a) Determine the center of the power series.

(b) Determine the radius of convergence for the power series.

(c) Determine the interval of convergence for the power series.
Question 6. Let \( f(x) = \cos(x) \).

(a) Compute the Taylor polynomial \( P_2(x) \) of order 2 at \( a = 0 \) for \( f \).

(b) Evaluate \( P_2(\pi/2) \).

(c) Taylor’s theorem states that

\[
|f(x) - P_n(x)| \leq M \frac{|x - a|^{n+1}}{(n + 1)!},
\]

where \( |f^{(n+1)}(t)| \leq M \) for all \( t \) between \( x \) and \( a \).

Use this to estimate the error in approximating \( f(\pi/2) \) using \( P_2(\pi/2) \).
Question 7. Find the Cartesian equation for the following parametric equations.

\[ x = \frac{1}{2} \tan t, \quad y = \frac{1}{2} \sec t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2} \]
Question 8. Find the length of the curve given by the following parametric equations.

\[ x = \frac{1}{2} t^2, \quad y = \frac{1}{3} (2t + 1)^{3/2}, \quad 0 \leq t \leq 4 \]
Question 9. Find the angles $0 \leq \theta \leq 2\pi$ for which the line tangent to the polar spiral $r = e^{-\theta}$ is horizontal; i.e., where the slope of the tangent line is zero.
Question 10. Find the area inside the circle $r = 2 \cos \theta$ and outside the circle $r = \sqrt{3}$. Sketch the figure. Give an exact answer and show your work.