Math 165: Final Exam — Part 2
Fall 2014

This part of the exam has 5 problems. Each problem is worth 12 points.

Answer each question completely. Show all work. No credit is allowed for mere answers with no work shown. Show the steps of calculations. State the reasons that justify conclusions.

Question 1: ______

Question 2: ______

Question 3: ______

Question 4: ______

Question 5: ______

60 Total Points: ______
**Question 1.** Find the area of the region bounded by the graphs of

\[ y = x^2 - 4x - 6 \quad \text{and} \quad y = -x^2 + 4x + 4. \]
Question 2. A function $f$ has derivative

$$f''(x) = (x + 1)^4(x - 4)^3.$$ 

a. Find and classify all critical point(s) for $f$. Justify your answers. (6 points.)

b. Find all inflection point(s) for $f$. Justify your answers. (6 points.)
**Question 3.** A researcher in horticulture wants to fence in a test garden. The garden must have area 720 ft$^2$ and will be split into six sub-plots as shown in the figure below. The fence around the perimeter of the garden must be sturdy and tall, and costs 8 dollars per foot. The inner fencing need not be as strong or tall, and only costs 2 dollars per foot. Find the dimensions of the pen of minimal cost and give the minimal cost. (Be sure to use the first derivative test or the second derivative test to show that your solution does indeed give the minimum.)
Question 4. Two long straight roads, one running East-West, and the other running North-South meet at a point $O$. At 12:00 noon, bicyclist Bob is on the E-W road at a point 100 kilometers East of $O$, while runner Renee is at point $O$. At this time Bob starts riding towards point $O$ and Renee starts running North. Bob bikes at a constant rate of 30 kilometers/hour and Renee runs at a constant rate of 15 kilometers per hour.

How fast is the distance between them changing at 2:00 pm?
Question 5. The graph of a function $g(t)$, defined on the interval $[-2, 6]$, is shown below. A function $G(x)$ is defined for $-2 \leq x \leq 6$ by

$$G(x) = \int_1^x g(t) \, dt.$$ 

a. Find the values of $G(-2)$, $G(1)$, and $G(4)$. (3 points.)

b. On which intervals is $G(x)$ increasing? Decreasing? You must give good reasons. (3 points.)
c. On what intervals is \( G(x) \) concave up? Concave down? You must give good reasons. (3 points.)

d. On the axes provided on below and using the information found in parts a., b., c, sketch the graph of \( y = G(x) \). (3 points.)