This test is closed book and closed notes. No sophisticated calculator is allowed for this test. For full credit show all of your work (legibly!). If you know about L’Hospital’s method, then do not use it to answer any question; if you don’t know about L’Hospital’s method, then you should definitely not use it to answer any question! Each problem is worth 10 points (a total of 50 points). Failure to circle your correct section will result in a 2 point deduction.

1. Suppose we have that \( f(4) = 3 \) and
   - the average rate of change of \( f(x) \) for \( x \) between 2 and 6 is 7;
   - the average rate of change of \( f(x) \) for \( x \) between 4 and 7 is 3;
   - the average rate of change of \( f(x) \) for \( x \) between 0 and 6 is 4;
   - the average rate of change of \( f(x) \) for \( x \) between 2 and 7 is 5.

Determine \( f(0) \).
2. Find $a$ and $b$ so that $f(x)$ is continuous at $x = 1$ given that

$$f(x) = \begin{cases} \frac{\sqrt{x + 3} - ax}{x - 1} & \text{if } x < 1; \\ \frac{5}{4}x + b & \text{if } x \geq 1. \end{cases}$$
3. Suppose that $f(x)$ is as shown in the figure below.

(a) List and classify all discontinuities for $0 \leq x \leq 4$ for $f(x)$.

(b) Find $\lim_{x \to 0} f\left(\frac{\sin x}{x}\right)$ or explain why the limit does not exist.
4. Find \( \lim_{{x \to 5}} \frac{\sqrt{x - 1} - \sqrt[4]{2x + 6}}{\sin(x - 5)} \).

(Note \( \sqrt[4]{t} \) is the fourth root of \( t \) and can also be thought of as \( \sqrt{\sqrt{t}} \).)
5. Using *any* method you like determine $g'(0)$ where $g(x)$ is defined piecewise as follows:

$$
g(x) = \begin{cases} 
x^2 \sin \left( \frac{1}{x} \right) & \text{if } x \neq 0; \\
0 & \text{if } x = 0.
\end{cases}$$

Make sure to explain your answer and what tools you used. (The function $g(x)$ is continuous; you do not need to worry about continuity.)