Quiz problem bank

Quiz 1 problems

1. Find all solutions \((x, y)\) to the following:
   \[
   xy - x + 2y = 2 \\
   x^2 + 5x + 4y = 0
   \]

2. Let \(g(x) = \ln \left( \frac{x^2}{\sin x} \right)\). Find \(g'(x)\).

3. Find the tangent line to \(f(x) = xe^x\) at \(x = 2\).

4. Let \(h(x) = 2x^3 - 9x^2 + 12x - \pi^2\). Find the critical points of \(h\) and classify them as local max, local min, or neither.

5. Find the second order Taylor polynomial of the function \(f(x) = \arctan x\) centered around \(x = 0\).

6. Find \(\int \frac{x^2 + 3x - 2}{x} \, dx\).

7. Find \(\frac{d}{dx} \left( \int_{\sqrt{x}}^{x^2} \sin(t^2) \, dt \right)\).

8. Find \(\int \frac{2}{e^x + 2} \, dx\).

9. Identify and sketch the curve \(r = 2\sin \theta\).

Quiz 2 problems

1. Find the volume for the set of points \((x, y, z)\) satisfying \(2y + 2z - 1 \leq x^2 + y^2 + z^2 \leq 2y + 3\).

2. Find the equation of the sphere which has the line segment joining \((-2, 3, 7)\) and \((4, -1, 5)\) as a diameter.

3. Rewrite \(\phi = \pi/6\), \(\theta = \pi/3\), \(\rho = \sqrt{6}\) as a point in Cartesian coordinates.

4. Describe, by picture and short explanation, the set of points satisfying \(\phi \geq \pi/4\) and \(\rho \leq 2\).

5. Rewrite \(\rho^2 = -\sec(2\phi)\) in Cartesian coordinates and cylindrical coordinates.

6. Rewrite \(\rho = \frac{1}{\cos \phi + \sin \phi}\) in Cylindrical coordinates and sketch the surface.

7. Write \(\phi = \frac{1}{4}\pi\) in cylindrical and Cartesian coordinates.

8. A 40 mph wind is blowing due south \((270^\circ)\), you are on a plane that has bearing of N 60\(^\circ\) E (aka 30\(^\circ\)), but the plane is traveling due east \((0^\circ)\). What is the airspeed of the plane? How fast is the plane moving relative to the ground?

9. Find the projection of \(\vec{u} = -i + 5j + 3k\) onto the vector \(\vec{v} = -i + j - 3k\).

Quiz 3 problems

1. Find the equation of the plane corresponding to the set of points \((x, y, z)\) equidistant from the points \((-2, 1, 4)\) and \((6, 3, -2)\).

2. Find \(\cos \theta\) where \(\theta\) is the angle between the planes \(x + 2y - 2z = 17\) and \(4x + 3z = 73\).

3. Which of the following do not make sense. (Explain your answer.)
   \[
   (\vec{u} \times \vec{v}) \cdot \vec{w}; \quad \vec{u} \times (\vec{v} \cdot \vec{w}); \quad (\vec{u} \cdot \vec{v})\vec{w}; \quad (\vec{u} \times \vec{v})\vec{w}
   \]

4. Find the plane which goes through the points \((2, 1, -3), (1, 0, 3)\) and \((1, 2, 1)\).

5. Find the area of the triangle with \((1, -1, -2), (-2, 0, -1)\) and \((0, -2, 1)\) as vertices.

6. Find the plane passing through the point \((-1, -2, 3)\) and perpendicular to both the planes \(x - 3y + 2z = 71\) and \(2x - 2y - z = e\).

7. While holding an egg you travel along
   \[
   \mathbf{r}(t) = (t^3 + t)\mathbf{i} + (6t^2 - 7)\mathbf{j} + (7 - t^3)\mathbf{k}.
   \]
   At time \(t = 1\) you let go of the egg. Determine where the egg will hit the xy-plane.

8. Find the line in parametric form containing the point \((0, 1, 2)\) and which perpendicularly intersects the line \(x = 1 + t, y = 1 - t, z = 2t\).

9. Let \(\mathbf{r}(t) = (2\cos(\pi t), \sin(\pi t), t^3)\). Find the plane perpendicular to this curve at time \(t = 2\).

Quiz 4 problems

1. A particle moving through three dimensional space has \(\mathbf{v}(t) = (\sec^2 t, 2\sec t \tan t, \tan^2 t)\) as its velocity function. If at \(t = 0\) the particle is at \((0, 1, 2)\) find the position of the particle at \(t = \frac{\pi}{4}\).

2. If \(\mathbf{F}(t) = \left\langle \arctan t, e^{t^2}, \int_0^t \sin(y^2) \, dy \right\rangle\), find \(\mathbf{F}'(t)\) and \(\mathbf{F}''(t)\).
3. Find \( \int_0^1 \left( e^t i + \cos(\pi t) j - \frac{t}{t^2 + 1} k \right) dt \).

4. Find the distance a particle travels along the curve \( r(t) = (\sin(2t), \frac{1}{2} t^{3/2}, \cos(2t)) \) from \( t = 0 \) to \( t = 5 \).

5. Find the distance a particle travels along the curve \( (e^t, e^{-t}, \sqrt{2} t) \) from \( t = 0 \) to \( t = 1 \).

6. Find curvature \( \kappa(t) \) for \( r(t) = (\sin t, \cos t, \frac{1}{2} t^2) \).

7. Find the curvature at time \( t = 0 \) for the curve \( r(t) = \left( 71 + \ln(t+1), \tan t + 2t^2 + \sqrt{\pi}, e^t \cos t - \frac{e^t}{6} \right) \).

---

**Quiz 5 problems**

1. Find \( a_T \) and \( a_N \) for \( r(t) = ti + t^2 j + \frac{2}{3} t^3 k \). Simplify your answers.

2. Find \( a_N \) for \( r(t) = (e^t + 17, 2e^{-t} - \pi, 137 - 2t) \).

3. Find all times \( t \) that the parametric curve given by \( (t^3 + e^t + 2, -t + \cos t + 1, 2t + e^t + 2 \cos t) \) intersects the plane \( x + 2y - z = 4 \).

4. Find \( (\sin^2 - 3z) + e^t + \sin(t)k \).

5. Give a formula for, and classify, the quadric surface containing the parametric curve \( (e^t (\sin t + \cos t), e^t, e^{2t} \sin(2t)) \).

6. Find the point on the plane \( x + 2y = 5 + 3z \) closest to the point \( (4, 4, -7) \). (Hint: the line between these two points is perpendicular to the plane.)

7. The following two lines are intersecting:

\[
\begin{align*}
  x &= 1 - t \\
  y &= 2 + t \\
  z &= 3 - 2t
\end{align*}
\]

Find the line which goes through the intersection point and is perpendicular to both given lines.

---

**Quiz 6 problems**

1. Sketch the domain of the following function:

\[
f(x, y) = \frac{\ln(x - y^2)}{\sqrt{4 - x^2 - y^2}}
\]

2. Describe the domain of the following function:

\[
g(x, y, z) = \frac{\sqrt{1 - x^2 - y^2 - z^2}}{\ln z}
\]

---

**Quiz 7 problems**

1. Given \( f(x, y, z) = x^2yz + y^2z^3 \) find \( \nabla f \).

2. Find all points \((x, y)\) so that \( \nabla f(x, y) = 0 \) where \( f(x, y) = x^2 - 6x + 2y^2 - 10y + 2xy + 137 \).

3. On the surface \( z = x^3 - 3xy + y^2 \), a marble is placed over the point \((1, 2)\). When released the marble initially move in the direction of steepest decrease. Find a vector pointing \((a, b, c)\) pointing in the direction the marble will initially move.

4. Find the directional derivative for the function \( f(x, y, z) = xz^2 - 3xyz + 2xyz - 3x + 5y - 17 \) from the point \((2, -6, 3)\) in the direction of the origin.

5. Let \( f(x, y, z) = x^2y^2z + 5z^2 \). Find a unit vector \( u \) in the direction in which \( f \) increases most rapidly at the point \( p = (-2, 1, -1) \), and find the rate of change of \( f \) in this direction.

6. Find an equation of the tangent plane to the surface \( x^2 + xy + y^2 + z^2 = 16 \) at the point \((1, 2, 3)\).

7. On the amazing can machine, which has the ability to alter the dimensions of cylindrical cans, the gauges currently read as follows: \( h = 5 \), \( \frac{dh}{dt} = -6\pi \), \( \frac{dr}{dt} = -\frac{1}{r} \), and \( \frac{dV}{tr} = 1 \), where \( V \), \( h \), \( r \) are volume, height and radius. But the gauge for \( r \) is broken. Find the possible value(s) for \( r \).
8. Given the implicitly relationship \( z + \sin z = xy \), find \( \frac{\partial^2 z}{\partial x \partial y} \) only in terms of \( z \).

9. Suppose that \( f(x, y) \) is differentiable and satisfies \( f(t^3 - t + 1, 2 - t^2) = t^4 - 4t^3 + 4t + 6 \). Find \( f_x(1, 1) \) and \( f_y(1, 1) \).

10. A machine is being set up to make cones to be used for storage. The volume of a cone is \( V = \frac{1}{3} \pi r^2 h \) where \( r \) is a radius and \( h \) is a height. The machine ideally makes the cones with \( r = 3 \) and \( h = 5 \) for a total volume of \( 15 \pi \) units however there tend to be slight imperfections. Given that the volume of a cone must be within \( 3\pi \) units (i.e., find the tolerance that we need to have for the radius (i.e., find \( \Delta r \)).

**Quiz 8 problems**

1. Find the second order Taylor polynomial for \( f(x, y) = e^{x+y^2} + x \sin y \) at the point \((0,0)\).

2. Find and classify the critical points for
\[
f(x, y) = x^3 - 8xy + 2y^2 - 3x + 4y - 23.
\]

3. Find and classify all of the critical points for the function \( f(x, y) = e^y(y^2 - x^2) \).

4. Find the critical points for
\[
g(x, y) = 2x^3 - 2x^2y + 6xy + y^2 - x^2 + 137
\]

5. Let \( h(x, y) = y^2 \sin x - x \). Verify that \((0,1)\) is a critical point and classify the point as a maximum, minimum or saddle.

6. The Nook corporation (maker of the finest Yops) has recently merged with the Jibboo conglomerate (maker of the finest Zans). Currently Yops sell for three dollars each and Zans sell for nine dollars each. By combining their production the new company enjoys economy of scope and is now able to produce \( y \) Yops and \( z \) Zans at a cost of \( 10 + \frac{1}{2}y^2 + \frac{1}{3}z^3 - yz \) dollars. Determine how many Yops and Zans respectively should be made in order to maximize profit. Also, verify that your answer is a maximum by using the second derivative test.

7. Use the technique of Lagrange multipliers to find the maximum value of \( f(x, y) = xy + y \) given that \( 9x^2 + 10y^4 = 9 \).

8. You have recently been hired to paint three large non-overlapping dots (a red dot, a blue dot, and a green dot) on the side of a building. They have left the design of the dots to you, the only instruction they gave is that the total area must be \( 200 \pi \) m\(^2\). The contract allows you to charge \( 3\pi \) thousand dollars for painting a red circle of radius \( r \), while for a blue circle you charge \( 4\pi \) thousand and for a green circle you charge \( 5\pi \) thousand. What radius should you choose for the various circles to maximize how much you charge?

**Quiz 9 problems**

1. Approximate the volume of the solid above the region \( R \) in the xy-plane consisting of the points \( 1 \leq x \leq 5 \) and \( 1 \leq y \leq 5 \), and below the surface \( f(x, y) = \frac{1}{\sqrt{x^2 + y^2}} \), by splitting \( R \) into four equally sized pieces and using the center point of each to approximate the height of each piece.

2. Let \( \int \int_{[a,b] \times [c,d]} f(x,y) \, dA \) denote the integral of \( f(x,y) \) over the region with \( a \leq x \leq b \) and \( c \leq y \leq d \). Find \( \int \int_{[2,3] \times [0,1]} f(x,y) \, dA \) given the following:
\[
\begin{align*}
\int \int_{[0,1] \times [0,2]} f(x,y) \, dA &= 4, \\
\int \int_{[0,2] \times [0,1]} f(x,y) \, dA &= -2, \\
\int \int_{[0,3] \times [1,2]} f(x,y) \, dA &= 3, \\
\int \int_{[1,3] \times [0,2]} f(x,y) \, dA &= 7.
\end{align*}
\]

3. Let \( R_1 \) be the region with \( 0 \leq x \leq 2 \) and \( 1 \leq y \leq 3 \), let \( R_2 \) be the region with \( 2 \leq x \leq 5 \) and \( 1 \leq y \leq 3 \), and let \( R \) be the union of the two regions. Given that \( \int_{R_1} g(x,y) \, dA = 8 \) and \( \int_{R} g(x,y) \, dA = 3 \) find \( \int_{R_2} (g(x,y) + 2) \, dA \).

4. Find
\[
\int_0^1 \int_0^{e^{-x} + x^2} \frac{y}{x + 2} \, dy \, dx.
\]

5. Find
\[
\int_0^2 \int_0^{137} (xe^{y^3} - e^{y^3}) \, dy \, dx.
\]

6. Find \( \int_R \frac{2}{1 + x^2} \, dA \) where \( R \) is the triangular region with vertices at \((0,0)\), \((2,0)\) and \((2,2)\).

7. Let \( R \) be the bounded region between \( y = x \) and \( y = x^2 \). Write \( \int_R f(x,y) \, dA \) as an iterated integral in both ways, i.e., \( dx \, dy \) and \( dy \, dx \).

8. By changing the order of integration write the following as a single integral
\[
\int_{-1}^{3} f(x,y) \, dx \, dy + \int_{0}^{1} f(x,y) \, dy \, dx + \int_{2}^{3} f(x,y) \, dy \, dx.
\]
9. Find \( \int_0^4 \int_{\sqrt{2}/2}^2 \sin(x^2) \, dx \, dy \).

10. Find \( \int_0^1 \int_0^{2x-x^2} \frac{1}{\sqrt{x^2+y^2}} \, dy \, dx \).

---

**Quiz 10 problems**

1. Find the volume of the set of points \( z \geq x^2 + y^2 \) and \( z \leq 12 - 2(x^2 + y^2) \).

2. Find the volume of the solid consisting of points above the xy-plane, below \( z = 10 - 2x + 3y \) and satisfying \( 1 \leq x^2 + y^2 \leq 4 \).

3. Find the surface area of the cone \( z^2 = x^2 + y^2 \) above the region \( R \) given by \( 0 \leq x \leq 137 \) and \( 0 \leq y \leq 1 \).

4. Set up (but do not evaluate) an integral for the surface area for \( f(x,y) = x^2y - y \) over the region \( 0 \leq x \leq 4 \) and \( 0 \leq y \leq 4 \).

5. Evaluate \( \int_S xy \, dA \) where \( S \) is the region in the first quadrant inside \( x^2 + y^2 = 9 \) and outside \( x^2 + y^2 = 4 \).

6. Find the volume of the solid region consisting of points \( x^2 \leq z \leq 4 - y^2 \).

7. Let \( S \) be the solid region consisting of points \( x^2 \leq z \leq 4 - y^2 \). Find \( \iiint_S (\sin(x^{137})e^z - y^3 \arctan(137z)) \, dV \).

8. Find \( \int_0^2 \int_{-x}^{1-x-y} e^{x-y} \, dz \, dy \).

9. Consider the following integral:
   \[
   \int_0^6 \int_0^{-2(1/3)x} \int_0^{2-(1/3)x-(1/2)y} f(x,y,z) \, dz \, dy \, dx
   \]
   Change the order of integration to \( dx \, dy \, dz \).

---

**Quiz 11 problems**

1. The Archimedean spiral is given by \( r = \theta \). Let \( R \) be the region between the origin and the Archimedean spiral for \( 0 \leq \theta \leq 2\pi \). Given that the density is thrice the distance to the origin, find the mass of the region.

2. Let \( R \) be the region \( x \leq 1 \) and \( x \geq y^2 \). Given that \( \delta(x,y) = xy^2 \) find \( (x, y) \).

3. Find the mass and center of mass of the region \( R \) consisting of the triangle with vertices \((0,0), (1,1)\) and \((-1,1)\) with \( \delta(x,y) = y \).

4. Convert the integral \( \int_0^\pi \int_0^{\sqrt{3}\pi} \int_0^{\sqrt{3}\pi} r^2 \sin\theta \, dz \, d\theta \) from cylindrical coordinates to Cartesian coordinates.

5. Convert the integral \( \int_0^\pi \int_0^{\sqrt{3}\pi} \int_0^{\sqrt{3}\pi} r^2 \sin\theta \, dz \, d\theta \) from cylindrical coordinates to spherical coordinates.

6. Set up, but do not evaluate, an integral to calculate the volume of the region inside the surfaces \( x^2 + y^2 + z^2 = 9 \), below the surface \( z = \sqrt{x^2 + y^2} \) and above the plane \( z = 0 \).

7. The Archimedean Spiral is the curve \( r = \theta \). For the homogeneous volume (i.e., density is constant) bounded below by the \( x\)-plane, bounded above by the surface \( z = x^2 + y^2 \) and over the region from the origin to the Archimedian Spiral for \( 0 \leq \theta \leq \pi \) find \( z \).

8. Let \( R \) be the region in the first quadrant bounded by the curves \( x = y^2 \), \( x = y^2 - 4 \), \( x = 9 - y^2 \) and \( x = 16 - y^2 \). Making the substitutions \( u = x - y^2 \) and \( v = x + y^2 \) describe this region in terms of \( u \) and \( v \) and determine \( J(u,v) \).

9. Evaluate the integral \( \int_0^1 \int_{1-y}^{1+y} ye^{\sqrt{x^2+y}} \, dx \, dy \) by making the change of variables \( u = \sqrt{x+y} \) and \( v = y \).

10. Find
    \[
    \int_0^2 \int_{x^2}^{2-x^2} 6x \cos((x^2 + y^3)) \, dy \, dx
    \]
    by making the substitutions \( u = x^2 + y \) and \( v = x \).

---

**Quiz 12 problems**

1. Find \( I_z \) for the region between the origin and the Archimedian spiral, \( r = \theta \), where \( 0 \leq \theta \leq \pi \).

2. Find \( I_z \) where \( \delta(x,y) = x + y^2 \) for the region \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \).

3. Let \( R \) be the set of points with \( 1 \leq x \leq 2 \) and \( 0 \leq y \leq 1 \) and \( \delta(x,y) = x \). Find the inertia when \( R \) is rotated around the line \( x = y \). (The distance between \( (a,b) \) and the line \( x = y \) is \( |a - b|/\sqrt{2} \)).

4. Find the mass of the cube \( 0 \leq x \leq 2 \), \( 0 \leq y \leq 2 \), \( 0 \leq z \leq 2 \) given that \( \delta(x,y,z) = xy + xz + yz \).

5. Evaluate the integral
    \[
    \iint (x-y) \sin(x+y) \, dx \, dy
    \]
    over the tilted square region with corners at \((0,0), (\pi/2, \pi/2), (\pi,0)\) and \((\pi/2, -\pi/2)\) by doing a change of variables \( u = x+y \) and \( v = x-y \).
6. Given a wire which is bent in the shape of a helix 
\( (\sin(3t), \cos(3t), 4t) \) for \( 0 \leq t \leq 2\pi \) and where the 
density of the wire is \( \delta(x, y, z) = z \), find the 
mass of the wire.

7. Evaluate \( \int_C (x^3 + y) \, ds \) where \( C \) is the curve given 
by \( x = 3t \) and \( y = t^3 \) for \( 0 \leq t \leq 1 \).

8. Let \( C = (\sin t, \cos t) \) for \( 0 \leq t \leq \pi \). Find
\[
\int_C x \, dx + x^2 y \, dy.
\]

---

### Quiz 13 problems

1. Given \( F = (y \sin x, x^2 + 2z, xy + 137) \), find \( \text{div} \, F \).

2. Given \( F = (y \sin x, x^2 + 2z, xy + 137) \), find \( \text{curl} \, F \).

3. Find the work done by moving a particle along 
the curve \( C \) given that \( F = (x^3 - y^4, xy^2) \) and the 
curve is \( x = t^2 \) and \( y = t^3 \) for \( -1 \leq t \leq 0 \).

4. Determine if the following vector valued function is 
conservative. If so find an \( f \) so that \( F = \nabla f \).
\[
F = \langle 3x^2 y^2 + e^z, 2x^3 y + \sin y \rangle
\]

5. Show the following vector valued function is 
conservative and find an \( f \) so that \( F = \nabla f \).
\[
F = \langle 2xyz + 3x^2 z + e^x, x^2 z + z, x^2 y + x^3 + \frac{1}{1+z^2} \rangle
\]

6. Evaluate the line integral
\[
\int_C yz \, dx + xz \, dy + (xy + 2z) \, dz
\]

where \( C \) is \( (\cos t, \sin t, t) \) for \( 0 \leq t \leq 2\pi \).

7. Evaluate the line integral
\[
\int_C (y + 2x) \, dx + (x - 2y) \, dy
\]

where \( C \) is the curve \( y = x^{137} \) with \( 0 \leq x \leq 1 \).

8. Let \( C \) be the curve that travels on the unit circle 
clockwise from \( (1,0) \) to \( (0,1) \). Calculate
\[
\int_C (2xe^y \cos(x^2)) \, dx + (e^y \sin(x^2) + x) \, dy.
\]

9. Evaluate the line integral
\[
\int_C 2xy \, dx + x^2 \, dy
\]

where \( C \) is the curve on the cardioid \( r = 2 + \cos \theta \) 
traveled counterclockwise.

10. Let \( R \) be the annulus consisting \( 1 \leq x^2 + y^2 \leq 4 \) 
and let \( C \) be the boundaries of this region 
oriented so that the region is always on the left 
while traveled along \( C \). Find \( \oint_C x \, dy \).

---

### Quiz 14 problems

1. Let \( G \) be the surface corresponding to the 
portion of the sphere \( x^2 + y^2 + (z - 2)^2 = 4 \) below 
the plane \( z = 2 \). Find
\[
\oint_G \langle xy^2 - 2y, xz + x^2 y, x^2 + y^2 \rangle \cdot T \, ds
\]

where the orientation on the boundary is 
clockwise when viewed from above.

2. Calculate
\[
\iint_G \text{curl}(F) \cdot n \, d(SA)
\]

where \( F = yz \mathbf{i} + 4xz \mathbf{j} + 2xz \mathbf{k} \) and \( G \) is the part 
of the sphere \( x^2 + y^2 + z^2 = 9 \) below the plane 
\( z = 2 \), and \( n \) is the outward 
pointing normal.

3. Let \( G \) be the surface \( z = x^2 - y^2 \) intersecting the 
(solid) cylinder \( x^2 - y^2 \leq 1 \). Find
\[
\oint_{\partial G} \langle x, x + y^2, x^2 + y^2 - z \rangle \cdot T \, ds
\]

where \( \partial G \) is oriented counter-clockwise when 
viewed from above.

4. Let \( S \) be the solid cube with one corner at \( (0,0,0) \) 
and the opposite corner at \( (1,1,1) \). Given that
\[
F = \langle x^3 y, \sin(z^2) - 2yz, z^2 + 137x^6 \rangle
\]

find
\[
\iiint_{\partial S} F \cdot n \, d(SA).
\]

5. Let \( S \) be the solid cone consisting of the points 
below the plane \( z = 4 \) and above \( z = \sqrt{x^2 + y^2} \). Find
\[
\iiint_{\partial S} F \cdot n \, d(SA),
\]

where
\[
F = \langle xy^2 + e^{y+\cos(y)}, x^2 y + \sin(z), z^2 + \cos x \rangle
\]

6. Let \( S \) be the solid consisting of the set of points 
satisfying \( 1 \leq x^2 + y^2 \leq 4 \) and \( 0 \leq z \leq 3 \). Find
\[
\iiint_{\partial S} \langle e^y - 2xz, z^4 - 2y - \sin(e^x), z^2 + 3z - \ln(1+x^2) \rangle \cdot n \, d(SA)
\]

7. [Mystery problem]