Functions of several variables

\[ Z = f(x,y) \quad \text{(two inputs)} \]
\[ \omega = g(x,y,z) \quad \text{(three inputs)} \]

Example: \[ Z = \frac{x^2 + y^2}{f(x,y)} \]

\[ f(1,1) = Z \]
\[ f(5,t) = s^2 + t^2 \]
\[ f(\text{cat, dog}) = (\text{cat})^2 + (\text{dog})^2 \]
Range = possible outputs
   (hard, need calculus)

Domain = possible inputs
   Usually need to figure out where we don’t have one of the following problems:

1. \( \frac{1}{0} \) (Division by 0)
2. \( \sqrt{<0} \) (square root of negative)
3. \( \ln(\leq 0) \) (log of a non-positive)
Example: Find domain of

\[ f(x,y) = \frac{\sqrt{y-x^2}}{x^2+(y-1)^2} \]

\[ y \geq x^2 \]
\[ x^2+(y-1)^2 \neq 0 \]

\( (0,1) \)

All \((x,y)\) so that \( y \geq x^2 \) and \((x,y) \neq (0,1)\)
Example: Find domain of

\[ g(x, y) = \frac{\sqrt{2x-x^2}}{\ln(x-y^2)} \]

- \[ 2x-x^2 \geq 0 \]
- \[ 0 \leq x \leq 2 \]
- \[ y = 2x-x^2 \]
- \[ y = x(2-x) \]
- \[ x-y^2 > 0 \]
- \[ x > y^2 \]
- \[ x-y^2 \neq 1 \]
- \[ x = y^2 + 1 \]

Graph showing the domain with values:
- \[ x = 2 \]
Graph of $z = f(x,y)$
level curves

One way to understand a function is by looking at cross sections for fixed \( z \). These cross sections (contours) correspond to curves in the plane and a collection of these curves is a contour map.

- Weather maps
- Topographical maps
Reading contour plots gives us information about the behavior of the function.

Goal of contour maps is to store 3-D information in a 2-D setting.
Functions of three variables

\[ w = f(x, y, z) \]

- range = all outputs
- domain = all inputs
  (look for same problems)
- graph

(Just kidding! Can't draw these)
Level surfaces

\[ g(x, y, z) = K \]

**Example:** Find level surfaces of \( g(x, y, z) = z - x^2 - y^2 = K \)

\[ z = x^2 + y^2 + K \]

![Diagram of level surfaces with labeled k values]
Notation on sets

- Empty set

Interior point — some small ball around the point is completely inside

Boundary point — every ball around the point is both in and out

Open set — every point is interior

Closed set — every boundary point in set

Bounded set — set inside of some large ball