Calculus

Now in exciting 3D
Calculus

Arithmetic
We understand flat things very well...

but the world around us is not flat...

so we approximate things which are not flat by things that are flat.
Differential calculus

How is something changing?
Integral calculus

How much is the total?
Motivation for calculus rooted in geometry

Position in 2D

Cartesian coordinates

Polar coordinates

\[ x = r \cos \theta \]
\[ y = r \sin \theta \]
\[ x^2 + y^2 = r^2 \]
\[ \tan \theta = \frac{y}{x} \]
Position in 3D

- Z-axis
  - (0, 0, z)
  - (x, 0, z) - xz-plane (y = 0)
  - (x, y, z)
- YZ-plane (x = 0)
  - (0, y, z)
- Y-axis
  - (0, y, 0)
- XY-plane (z = 0)
  - (x, y, 0)
  - (x, 0, 0) - x-axis

Cartesian coordinates
\[ D^2 = C^2 + (z_2 - z_1)^2 \]
\[ = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \]
\[ D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \]

**Sphere** is the set of all points distance \( r \) away from center \((h, k, l)\).
Example:

\[ x^2 + 7x + y^2 + 8y + z^2 + 10z = 2 \]

\[
\begin{align*}
&x^2 + 7x + \frac{49}{4} + y^2 + 8y + 16 + z^2 + 10z + 25 = 2 + \frac{16}{4} + 25 \\
&(x + \frac{7}{2})^2 + (y + 4)^2 + (z + 5)^2 = \frac{221}{4}
\end{align*}
\]

Center: \((-\frac{7}{2}, -4, -5)\) \(r = \sqrt{\frac{221}{4}}\)