This test is closed book and closed notes. A (graphing) calculator is allowed for this test but cannot also be a communication device (i.e., your iPhone is not a calculator). Answer each question completely using exact values unless otherwise indicated. Show your work (legibly); answers without work and/or justifications will not receive credit. Place your final answer in the provided box. Each problem is worth 10 points for a total of 80 points.
1. (a) Find a direction vector for the line of intersection of the planes $x + 2y + z = 0$ and $x + y + 1 = 0$.

Answer:

(b) Find the equation of the plane containing the line $x = 2t$, $y = 1 - t$, $z = 3t$ and the vector found in part (a).

Answer:
2. A particle travels along the parametric curve

\[ x(t) = t + \ln t, \quad y(t) = t - \ln t \quad z(t) = 7 - 4\sqrt{t}. \]

Find the distance the particle travels in the time interval \(1 \leq t \leq 2\).
3. Let \( f(x, y, z) = x^2 y^3 z + 5z^2 \).

(a) Find a unit vector \( \mathbf{u} \) in the direction in which \( f \) increases most rapidly at the point \( \mathbf{p} = (-2, 1, -1) \).

Answer:

(b) What is the rate of change in this direction?

Answer:
4. Find and classify all of the critical points for the function

\[ f(x, y) = x^4 - 4xy - 7x^2 + 4y^2 + 4x - 8y + 20. \]
5. Evaluate

\[ \int \int_S xy \, dA, \]

where \( S \) is the region in the first quadrant inside \( x^2 + y^2 = 9 \) and outside \( x^2 + y^2 = 4 \).

Answer:
6. Find
\[
\int_0^2 \int_{x-x^2}^{2-x^2} 6x \cos ((x^2 + y)^3) \, dy \, dx
\]
by making the substitutions \( u = x^2 + y \) and \( v = x \).

Answer:
7. Let C be the closed curve which consists of straight line segments between the points \((-3,0), (3,0)\) and \((0,3)\) where C is oriented counterclockwise. Find

\[
\oint_C \left( y^2 + \cos(x^3) - x^2y \right) \, dx + \left( 2xy + \frac{1}{1 + e^{2y}} \right) \, dy.
\]

Answer: 

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8. Let $S$ be the sphere of radius 2 and $\mathbf{F}(x, y, z) = (3xy^2 + e^z y, 3x^2 y - 3 \cos(zx^3), z^3 + 17x^{25})$. Use Gauss’s Divergence Theorem to calculate

$$\iint_{\partial S} \mathbf{F} \cdot \mathbf{n} \, dS.$$

Answer: