1. For a graph $G$ the (multi-)set of degrees is called its degree sequence. Show that if there is a simple graph $G$ with $|E| \geq |V| - 1$, and no isolated vertices, then there is a simple graph $H$ which is connected and has the same degree sequence (so must also have the same number of edges as well as no isolated vertices).

2. Let $G$ be a $k$-regular bipartite graph with $k \geq 2$. Show that $\kappa(G) \neq 1$.

3. Let $G$ be a simple graph on $2n$ vertices with all degrees at least $n$. Show that $G$ has a 1-factor (i.e., a maximum matching with $n$ edges).

4. Let $M$ be a matching in a bipartite graph $G$. Show that if $M$ is sub-optimal, i.e., contains fewer edges than some other matching in $G$, then $G$ contains an augmenting path with respect to $M$. Does this generalize to matchings in non-bipartite graphs?