

QUIZ 2 385 FALL 2004

**Problem 1.** Show that the Sturm-Liouville problem

$$X'' + \lambda X = 0, \quad X'(0) = 0, \quad 3X(\log \frac{1}{2}) + 5X'(\log \frac{1}{2}) = 0$$

has a nontrivial solution for  $\lambda = -1$

**Problem 2.** Show that the problem

$$X'' + \lambda X = 0, \quad X(0) = 0, \quad 2X(1) = 3$$

has a solution for some  $\lambda < 0$ . Determine such values of  $\lambda$  and the corresponding solutions.

**Problem 3.** Solve the boundary value problem

$$\begin{aligned} u_t &= u_{xx} - hu \\ u_x(0, T) &= 0, \quad u(a, t) = T, \\ u(x, 0) &= \frac{T}{a^2} x^2 \end{aligned}$$

Here  $h$  and  $T$  are positive constants.

**Problem 4.** Show that solutions of the problem

$$X'' + \lambda X = 0, \quad X(0) = 0, \quad \lambda X(\pi) + X'(\pi) = 0$$

corresponding to different values of  $\lambda > 0$  are not orthogonal.