

EXTRA CREDIT HOMEWORK 1, COURSE 385, FALL 2004

Problem 1. Find a solution $u(x, t)$ to the boundary value problem

$$\begin{aligned}u_t &= ku_{xx} \quad 0 < x < 1, \\u(0, t) &= u(1, t) = 1 \\u(x, 0) &= 1 + \sin(\pi x)\cos(\pi x).\end{aligned}$$

Problem 2. Find a solution $u(x, t)$ to the boundary value problem

$$\begin{aligned}u_t &= ku_{xx} \quad 0 < x < 2, \\u(0, t) &= 0, \quad u(2, t) = 2 \\u(x, 0) &= x + \sin\left(\frac{\pi x}{2}\right).\end{aligned}$$

Problem 3. Find a Fourier series for the function $f(x) = e^{|x|}$, $-\pi < x < \pi$.

Problem 4. Find a steady-state solution to the boundary value problem

$$\begin{aligned}u_t &= [(1+x)u_x]_x + 2(1+x) \quad 0 < x < 9, \\u_x(0, t) &= 0, \quad u(9, t) = 300 \\u(x, 0) &= f(x).\end{aligned}$$

Problem 5. Find a steady-state solutions to the boundary value problem

$$\begin{aligned}u_t &= (e^{-x}u_x)_x \quad 0 < x < 10, \\u(0, t) - u_x(0, t) &= 320, \quad u(10, t) + u_x(10, t) = 320 \\u(x, 0) &= f(x).\end{aligned}$$