

EXAM 2 385 FALL 2004

Problem 1. Consider the problem

$$x^2 X'' + xX' + \lambda X = 0, \quad X(1) = 0, \quad X(e) = 0 \quad (1)$$

A) Find all possible solutions to problem (1).

B) Is this problem a **REGULAR** Sturm-Liouville problem?

Problem 2. Show that the eigenvalues of the Sturm-Liouville problem

$$x^2 X'' + xX' + \lambda X = 0, \quad X'(1) = 0, \quad X(b) + X'(b) = 0$$

are the roots of the equation

$$\sqrt{\lambda} \sin(\sqrt{\lambda} \ln b) = b \cos(\sqrt{\lambda} \ln b)$$

Problem 3. Solve the boundary value problem

$$u_t = u_{xx} - 2u_x \quad 0 < x < \pi$$

$$u(0, t) - u_x(0, t) = 0$$

$$u(\pi, t) + u_x(\pi, t) = 0$$

$$u(x, 0) = 1 + \sin(x)$$

Hint. Look for the function $u(x, t)$ in the form $u(x, t) = e^x w(x, t)$.

Problem 4. Solve the boundary value problem

$$e^{2x} u_t = (e^{2x} u_x)_x \quad 0 < x < \pi$$

$$u(0, t) = 0, \quad u(\pi, 0) = 0,$$

$$u(x, 0) = \sin(x)$$

Hint. Look for the function $u(x, t)$ in the form $u(x, t) = e^{-x} w(x, t)$.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$ - $\text{T}_{\text{E}}\text{X}$