

Exam 3.

Evaluate the line integral $\int_C (x^2 + y^2)dx + 2xydy$ where the curve C is the circle $x^2 + y^2 = 5$ oriented counterclockwise.

The surface S is represented by the function $\mathbf{r}(u, v) = (u + v)\vec{i} + (u - v)\vec{j} + \sin v\vec{k}$ where $0 \leq u \leq 2, 0 \leq v \leq 2\pi$. Find the unit normal vector to this surface at the point $(1, 1, 0)$ and the equation of a tangent plane to the surface at this point.

Given vector fields $\mathbf{F}(x, y, z) = y^5 \sin z\vec{i} + (y + \cos x)\vec{j} + z\vec{k}$, and $n(x, y, z) = \frac{1}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}(-x, -y, -z)$. Find the flux $\int_S \mathbf{F} \cdot dS$ where S is the unit sphere centered at origin.

Find the integral $\int_S \text{curl } \mathbf{F} \cdot dS$ where $\mathbf{F}(x, y, z) = (x^2 + y)\vec{i} + y^2\vec{j} + z\vec{k}$ and the surface $S = \{(x, y, z) | z = 0, x^2 + y^2 \leq 1\}$.